CMSC-37110 Discrete Mathematics
FIRST QUIZ October 6, 2011

Name (print): ____________________________________

Do **not** *use book, notes, scratch paper.* **Show all your work.** If you are not sure of the meaning of a problem, **ask the instructor.** The *bonus problems* are underrated, do not work on them until you are done with everything else. **Write your solution in the space provided.** You may CONTINUE ON THE REVERSE. This exam contributes 6% to your course grade.

All variables in the problems below are integers except where expressly stated otherwise.

1. (5+8 points) (a) State the prime property. Your definition should start with this: "We say that a number k has the prime property if. . . " Fill in the dots with a well quantified formula. (b) Prove that the prime numbers have the prime property. Use but do not prove the fact that $\gcd(ac, bc) = |c| \cdot \gcd(a, b)$.

2. (8 points) Find all integers z such that $z \mid z - 6$. Prove your answer.

3. (6+6 points) Let $a = 4k - 1$ and $b = 7k + 3$. Prove: (a) If $\gcd(a, b) \neq 1$ then $\gcd(a, b) = 19$. (b) There exist infinitely many values of k such that $\gcd(a, b) \neq 1$.

4. (4+4+5+4B points) Compute each multiplicative inverse or prove it does not exist; your answer x (if exists) should be in the range $0 \le x < m$ where m is the modulus. k, x are positive integers. (a) $5^{-1} \pmod{51}$ (b) $21^{-1} \pmod{91}$ (c) $k^{-1} \pmod{k^2 + k + 1}$ (d) BONUS: $(x+1)^{-1} \pmod{x^2 + 1}$.

5. (7+7 points) Let a_n, b_n be sequences of positive real numbers. Assume $a_n \sim b_n$. Circle the correct answer. Prove.

   (a) Does it follow that $a_n^n \sim b_n^n$? **YES** **NO**

   (b) Does it follow that $a_n + 1 \sim b_n + 1$? **YES** **NO**

6. (BONUS PROBLEM: 12B points) Let p^k be a prime power divisor of $\binom{n}{k}$. Prove: $p^k \le n$.

7. (BONUS PROBLEM: 6B points) True or false? Circle the correct answer. Prove. **TRUE** **FALSE**

$$\left(1 + \frac{1}{n}\right)^{n^2} \sim \mathrm{e}^n.$$