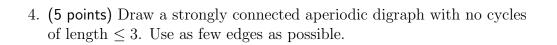
## CMSC-37110 Discrete Mathematics THIRD QUIZ November 29, 2011

Name (print):
Do not use book, notes, scratch paper. Show all your work. If you are no
sure of the meaning of a problem, ask the instructor. The bonus problem
are underrated, do not work on them until you are done with everything else
Write your solution in the space provided. You may CONTINUE OF
THE $\overline{\text{REVERSE}}$ . This exam contributes 6% to your course grade.
1. (12 points) We have a biased coin; the probability of "heads" is 1/3
Consider the experiment that we flip the coin $n$ times. We repeat thi
experiment $n^2$ times. Let $p(n)$ denote the probability that in each of th
experiments, the number of heads is between $0.33n$ and $0.34n$ . Prove
1 - p(n) is exponentially small.

2. (5 points) True or false: if A, B are  $2 \times 2$  real matrices then  $\det(A+B) = \det(A) + \det(B)$ . State and prove your answer.

3. (6+5 points) Let G be a 4-regular bipartite graph. ("4-regular" means every vertex has degree 4.) (a) Prove: G is not planar. (b) Draw a counterexample if we drop the condition that G is bipartite. Use as few vertices as possible.



5. (6 points) Prove: if a finite Markov chain has two stationary distributions then it has infinitely many.

6. (9 points) Determine the rank of the  $n \times n$  matrix  $B = (b_{i,j})$  where  $b_{i,j} = i + j$ . Prove your answer.

7. (12 points) Prove: if a digraph G is not strongly connected then it has a cut. (A cut is a partition  $V = A \dot{\cup} B$  of the set of vertices such that A, B are nonempty and there is no edge from B to A.)

8. (BONUS PROBLEM: 4B points) Prove: if A, B are  $n \times n$  real matrices then  $AB - BA \neq I$  (where I denotes the identity matrix).