CMSC-37110 Discrete Mathematics
FIRST QUIZ October 18, 2012

Name (print): ____________________________________

Do **not** *use book, notes, scratch paper.* **Show all your work.** If you are not sure of the meaning of a problem, **ask the instructor.** The *bonus problems* are underrated, do not work on them until you are done with everything else. **Write your solution in the space provided.** You may continue on the reverse. This exam contributes 6% to your course grade.

1. (9 points) Let $E(n)$ denote the set of even subsets (subsets of even size) and $O(n)$ the set of odd subsets of a set of n elements. Give a simple bijection between these two sets (assuming $n \geq 1$).

2. (9 points) Prove by induction on k: $(\forall x)(\forall k \geq 1)($ if x is odd then $x^{2^k} \equiv 1 \pmod{2^{k+2}})$.

3. (3+6 points) **True** or **false** (circle one, prove). All quantifiers range over the integers. Prove your answers.
(a) $(\forall x)(\exists y)(\gcd(x, y) = x - y)$ **T** **F**
(b) $(\forall x)(\exists y)(x^2 - y^2 \equiv 1 \pmod 7)$ **T** **F**

4. (3+3+3+5 points (5 for the proof(s))) **True** or **false** (circle one; prove if your anser is "False"). Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive

numbers. Suppose $a_n \sim b_n$. Does it follow that
(a) $a_n^2 \sim b_n^2$ **T** **F**
(b) $\sqrt{a_n} \sim \sqrt{b_n}$ **T** **F**
(c) $2^{a_n} \sim 2^{b_n}$ **T** **F**

5. (11 points) Let $a = 7k + 2$ and $b = 9k - 5$. Prove: $\gcd(a, b)$ is either 1 or 53.

6. (8 points) Decide, for what values of k does $(k + 1)^{-1} \pmod{k^2 + 1}$ exist, and for those values, find it (between 0 and k^2).

7. (BONUS: 5B points) Suppose P is the product of k consecutive integers ($k \geq 1$). Prove: $k! \mid P$.

8. (BONUS: 7B points) Let $S(n, 3) = \sum_{k=0}^{\lfloor n/3 \rfloor} \binom{n}{3k}$.
Prove: $|S(n, 3) - 2^n/3| < 1$.