Name: ________________________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse.** When describing an algorithm in pseudocode, **explain the meaning of your variables** (in English). This quiz contributes 4% to your course grade.

1. **(20 points)** Given an \( n \)-bit positive integer \( x \) (its bit-length in binary is \( n \)), determine \( \lfloor \sqrt{x} \rfloor \) in polynomial time. Write your algorithm in pseudocode. Minimize the number of comparisons made by the algorithm. Show that the number of comparisons is \( \sim cn \) for some constant \( c \); determine \( c \). Show that the overall complexity of the algorithm (number of bit operations) is \( O(n^C) \); find the smallest value of \( C \) assuming multiplication of two \( n \)-digit integers is performed by the schoolbook algorithm and thus takes \( \Theta(n^2) \).
2. (2+3+15 points) Describe all algorithms in pseudocode. **Explain the meaning of your variables.** Let $G$ be a digraph given by an array of adjacency lists; the vertex set is $V = \{1, \ldots, n\}$. “Linear time” means $O(n + m)$ where $m$ is the number of edges.

(a) Determine the in-degree of every vertex in linear time. The output of your algorithm should be the array of in-degrees.

(b) If every vertex has in-degree $\geq 1$ then find a directed cycle in $G$ in linear time.

(c) Recall that a topological sort of $G$ is a labeling of the vertices $1, \ldots, n$ such that if $i \rightarrow j$ is an edge then $i < j$. Give a linear-time algorithm that either finds a directed cycle in $G$ or topologically sorts $G$. 