Algorithms CMSC-37000 First Quiz. February 20, 2014
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Name: _____

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse.** When describing an algorithm in pseudocode, **explain the meaning of your variables** (in English). This quiz contributes 4% to your course grade.

1. (20 points) Given an n-bit positive integer x (its bit-length in binary is n), determine $\lfloor \sqrt{x} \rfloor$ in polynomial time. Write your algorithm in pseudocode. Minimize the number of comparisons made by the algorithm. Show that the number of comparisons is $\sim cn$ for some constant c; determine c. Show that the overall complexity of the algorithm (number of bit operations) is $O(n^C)$; find the smallest value of C assuming multiplication of two n-digit integers is performed by the schoolbook algorithm and thus takes $\Theta(n^2)$.

2. (2+3+15 points) Describe all algorithms in pseudocode. **Explain the meaning of your variables.** Let G be a digraph given by an array of adjacency lists; the vertex set is $V = \{1, \ldots, n\}$. "Linear time" means $O(n + m)$ where m is the number of edges.

   (a) Determine the in-degree of every vertex in linear time. The output of your algorithm should be the array of in-degrees.

   (b) If every vertex has in-degree ≥ 1 then find a directed cycle in G in linear time.

   (c) Recall that a *topological sort* of G is a labeling of the vertices $1, \ldots, n$ such that if $i \to j$ is an edge then $i < j$. Give a linear-time algorithm that either finds a directed cycle in G or topologically sorts G.