1. (16 points) The MAX-ACYCLIC-SUBGRAPH problem takes a directed graph $G = (V, E)$ as input and asks to find a largest acyclic subset $F \subseteq E$ (i.e., $(V, F)$ must be a DAG (directed acyclic graph)). This problem is NP-hard. Find a factor-2 approximation, i.e., an acyclic subset $F' \subseteq E$ that is at least half as large as the largest acyclic subset. Find $F'$ in linear time. Assume $V = \{1, 2, \ldots, n\}$. (The solution is very simple.) Describe your algorithm in pseudocode. Reason why your solution is within the required factor of the optimum.

2. (6 points) Recall that ILP ("feasible integer linear programs") is the language that consists of those pairs $(A, b)$ where $A \in \mathbb{Z}^{k \times n}$ is a $k \times n$ integer matrix (all entries are integers) and $b \in \mathbb{Z}^k$ a column vector with $k$ coordinates that are integers, such that the system $Ax \leq b$ of linear inequalities has an integer solution $x \in \mathbb{Z}^n$. ($k$ and $n$ are input variables, they are not fixed.) It is a fact that ILP $\in$ NP. Explain why this fact is difficult to prove. What would be a witness? Make a mathematical statement that we would need to prove: “If an integer solution exists then . . .”
3. (18 points) (Bellman-Ford algorithm): Let $G = (V, E, w, s)$ be a digraph with weighted edges and root $s \in V$. The weight function is $w : E \to \mathbb{R}$. Negative weights are permitted but no negative cycles. For each vertex $t \in V$ simultaneously, find the minimum cost of reaching $t$ from $s$. Describe your algorithm in pseudocode. The algorithm should run in $O(mn)$ where $m$ is the number of edges and $n$ the number of vertices. The weights are real numbers; the cost of an arithmetic operation on reals is one unit. (Hint: dynamic programming. A clear definition of your variables (the “brain” of the algorithm) accounts for half the credit.)