Honors Combinatorics CMSC-27410/Math-28400/CMSC-37200 First Quiz. April 4, 2014
Instructor: László Babai

Name (print): ________________________________ **Major/Year** ____________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse.** This quiz contributes 3% to your course grade.

1. (4 points) Recall: a *Sperner family* on n elements is a set of pairwise incomparable subsets of $[n] = \{1, \ldots, n\}$. ("Incomparable" means neither is a subset of the other.) True or false: every *maximal* Sperner family on n elements is *maximum*. Prove your answer.

2. (4 points) Recall: a *hypergraph* on the vertex set V is a set of subsets of V. Count the hypergraphs on a given set V of n vertices. Give a simple closed-form expression. Do not prove.

3. (4 points) Recall: the *generating function* of the sequence $a_0, a_1, \ldots$ is the function $f(x) = \sum_{n=0}^{\infty} a_n x^n$. Give closed-form expressions of the generating functions of the sequences
(a) $a_n = 1/(n+1)$ (b) $b_n = \binom{100}{n} 2^{-n}$

4. (3 points) Recall: $\pi(x)$ denotes the number of primes $\leq x$. State the Prime Number Theorem. Define the symbol involved in the statement.

5. (15 points) Prove the BLYM inequality (called LYM(B) in last class): If $\{A_1, \ldots, A_m\}$ is a Sperner family of subsets of $[n]$ then

$$\sum_{i=1}^{m} \frac{1}{\binom{n}{|A_i|}} \leq 1.$$