

Honors Combinatorics CMSC-27410/Math-28400/CMSC-37200 Second Quiz. April 14,
2014

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Name (print): _____ Major/Year _____

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse**. This quiz contributes 6% to your course grade.

1. (5 points) We roll n dice. What is the expected value of the product of the numbers shown? Your answer should be a simple closed-form expression. Do not prove.
2. (3+8 points) Consider a random graph G on a given set of n vertices; for each pair of vertices, we decide adjacency by flipping a coin. (a) What is the size of the sample space for this experiment? (b) What is the expected number of cliques of size t ? (Give a simple closed-form expression.) Prove your answer.
3. (14 points) Prove: if G is a triangle-free graph then $\chi(G) \leq 1 + 2\sqrt{n}$. (n is the number of vertices.)

4. (3+3+7 points) Let $\mathcal{H} = (V, \mathcal{E})$ be a hypergraph.
- (a) Define what is an independent set in \mathcal{H} .
 - (b) Define what is a legal coloring of \mathcal{H} .
 - (c) Prove:

$$\alpha(\mathcal{H})\chi(\mathcal{H}) \geq n.$$

Here n is the number of vertices, α the independence number (size of largest independent subset), and χ the chromatic number.

5. (17 points) Prove the Mantel–Turán theorem: A triangle-free graph on n vertices has at most $n^2/4$ edges.