Honors Combinatorics CMSC-27410/Math-28400/CMSC-37200 Fourth Quiz. May 5, 2014
Instructor: László Babai

Name (print): ______________________________ **Major/Year** ____________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse.** This quiz contributes 6% to your course grade.

1. (12 points) Recall from class that for almost all graphs G we have $\alpha(G) < 1 + 2\log_2 n$. Use this to prove: For almost all graphs G we have $\chi(G) > \omega(G)^{100}$.
(Notation: $\omega(G)$ is the clique number of G (size of largest clique).)

2. (12 points) For what primes p is the ring $\mathbb{F}_p[i]$ a field, where $i^2 = -1$? Outline the proof.

3. (12 points) Let $p \equiv -1 \pmod 4$ be a prime. Prove that the Paley tournament $P(p)$ is
(a) vertex-transitive (b) edge-transitive.
Explanation: The vertex set of $P(p)$ is $V = \{0, 1, \ldots, p-1\}$ where we have an edge $a \to b$ exactly if $b - a$ is a quadratic residue mod p. (a) means all vertices are equivalent under automorphisms, i.e., $(\forall a, b \in V)(\exists g \in \mathrm{Aut}(P(p)))(g(a) = b)$. (b) means $(\forall a, b, c, d \in V)($ if there is an $a \to b$ edge and a $c \to d$ edge then $\exists g \in \mathrm{Aut}(P(p)))(g(a) = c$ and $g(b) = d)$.

4. (24 points) Prove: If the graph G is vertex-transitive then $\alpha(G)\chi(G) \le n(1 + \ln n)$.
($\alpha(G)$ is the indepedence number, $\chi(G)$ the chromatic number, n the number of vertices.)