Honors Combinatorics CMSC-27410/Math-28400/CMSC-37200 Fifth Quiz. May 14, 2014
Instructor: László Babai

Name (print): ______________________________ **Major/Year** ____________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse.** This quiz contributes 6% to your course grade.

1. (8 points) Recall that a Steiner Triple System (STS) is an incidence geometry satisfying the axioms that (1) every line has 3 points (2) for every pair of distinct points x, y there is exactly one line passing through x and y. Prove: If a STS has n points then $n \equiv 1$ or $3 \pmod 6$.

2. (6+6+24 points) Let M be the incidence matrix of a projecive plane of order n. (This is an $(n^2+n+1) \times (n^2+n+1)$ matrix whose rows correspond to the points, columns to the lines, and the entries are $0, 1$, where the 1s indicate incidence.) (a) State the matrix equation that captures the axioms of projective planes. (b) Prove that $\lambda_1 = n + 1$ is an eigenvalue of M. Find a corresponding eigenvector. (c) Prove: all the $n^2 + n$ other (complex) eigenvalues have absolute value $\sqrt{n}$.

3. (16 points) Prove: If the graph G is vertex-transitive then $\alpha(G)\chi(G) \leq n(1 + \ln n)$.
   ($\alpha(G)$ is the indepedence number, $\chi(G)$ the chromatic number, n the number of vertices.)

4. (BONUS: 8 points) Construct 3 orthogonal Latin squares of order 4. (Your answer should be three 4×4 matrices.)