Theorems proved in class or in homework

LIST INCOMPLETE, PLEASE CHECK LATER
Last updated: 5-28 noon

Notation:
Good characterizations: (GC)
Counting: (CT)
Probabilistic method: (PR)

Matchings

- König's Theorem: max matching = min cover in bipartite graphs (GC)
- König's–Hall Marriage Theorem (GC)
- Tutte's Theorem: existence of perfect matchings in graphs (GC)
- Petersen's Theorem: bridgeless trivalent graph has perfect matching

Network flows and graph connectivity

- Max-flow min-cut Theorem (Ford–Fulkerson) (GC)
- Menger's Theorems: 4 versions (GC)

Independence number, chromatic number

- Bipartite iff no odd cyle (GC)
- Chromatic polynomial is a polynomial (CT)
- Stanley's Theorem: chromatic polynomial vs. acyclic orientations (CT)
- directed line-graph
- triangle-free graph with large chromatic number
- $\chi(G) \geq n/\alpha(G)$
- Erdős: for almost all graphs, $\alpha(G) < 1 + 2\log_2 n$
- Erdős: graphs with large girth and large chromatic number (PR)

Ramsey theory

- $9 \to (4,3) \quad 17 \to (3,3,3)$
- Erdős–Szekeres: $\binom{k+\ell}{k} \to (k+1, \ell+1)$
- $n \to ((1/2)\log_2 n, (1/2)\log_2 n)$
- explicit Ramsey graphs: $n \not\to 1+\sqrt{n}, 1+\sqrt{n})$
- Z. Nagy's explicit Ramsey graphs: $n \not\to (cn^{1/3}, cn^{1/3})$
- Ramsey's Theorem for colorings of the complete graph
- Ramsey's Theorem for colorings of the complete k-uniform hypergraphs (coloring the set of k-tuples of vertices) (stated)
- Van der Waerden's, Szemerédi's Theorem, Green–Tao Theorem (stated)

Digraphs

- DAG iff exists topological sort (GC)
- strong components
- If Eulerian and weakly connected then strongly connected
- tournaments: Hamilton path; if strongly connect then Hamilton cycle
- Erdős: existence of k-paradoxical tournaments (PR)

Trees

- exists vertex of degree 1; $m = n - 1$
- Cayley's formula (CT)

Planar graphs

- Jordan curve Theorem (stated)
- Euler's formula for connected plane multigraphs: $n - m + r = 2$
- For connected planar graphs, if $n \geq 3$ then $m \leq 3n - 6$

- For triangle-free connected planar graphs, if $n \geq 3$ then $m \leq 2n - 4$
- K_5 and $K_{3,3}$ are not planar
- topological graphs, Kuratowski graphs, Kuratowski's Theorem (GC) (stated)

Extremal graph theory

- Mantel–Turán's Theorem (max number of edges in triangle-free graph)
- Turán's Theorem (max number of edges in K_r-free graph)
- Kőváry–Turán–Sós Theorem (max number of edges in C_4-free graph and in $K_{2,3}$-free graph

Finite probability spaces
Random graphs

- Almost all graphs have dianeter 2
- Erdős: for almost all graphs, $\alpha(G) < 1 + 2\log_2 n$
- For almost all graphs, $\chi(G) > n/(1 + 2\log_2 n)$
- The Erdős–Rényi model $\mathbf{G}_{n,p}$
- Expected number of k-cycles in $G \in \mathbf{G}_{n,p}$