11.1 DO: Review the the quiz problems (posted).

11.2 DO: Assume the graph $G$ has minimum degree $\leq k$ hereditarily (i.e., every subgraph has minimum degree $\leq k$). Prove: $\chi(G) \leq k + 1$.

11.3 HW (6 points): Prove: If $G \not\cong K_{2,3}$ then $\chi(G) = O(\sqrt{n})$. – You may use other HW and DO exercises (present and past) without proof but with clear reference.
11.4 DO: Review the proof of the max-flow-min-cut theorem.

11.5 DO: Prove that the set of feasible flows is a (a) non-empty (b) convex (c) closed subset of \( \mathbb{R}^m \).

11.6 HW (7+5 points): We are given a network as in the preamble. Assume every \( s-t \) cut has infinite capacity. (a) Prove: there exists a directed \( s \to \cdots \to t \) path of which every edge has infinite capacity.
(b) Prove: the values of the \( s-t \) flows are unbounded (can be arbitrarily large).

11.7 DO: Prove: if an integral network has a cut of finite capacity then it has an optimum integral flow.

11.8 DO: State the four versions of Menger’s theorem.

11.9 DO: Deduce the directed edge-disjoint version of Menger’s Theorem from the max-flow-min-cut theorem and problem 11.7.

11.10 HW (8 points): Deduce König’s Theorem from the directed vertex-disjoint version of Menger’s Theorem.

11.11 DO: Deduce the directed vertex-disjoint version of Menger’s Theorem from the directed edge-disjoint version.

11.∞ DO: More problems may follow, please check back later.