Graph Theory – CMSC-27500 – Spring 2015
http://people.cs.uchicago.edu/~laci/15graphs
Homework set #7. Posted 4-22, 12:30am
Due Thursday, April 23, typeset in **LaTeX**.

Do not submit homework before its due date; it may get lost by the time we need to grade them. If you must submit early, write the early submissions on separate sheets, separately stapled; state "EARLY SUBMISSION" on the top, and send email to the instructor listing the problems you submitted early and the reason of early submission.

Read the homework instructions on the website. The instructions that follow here are only an incomplete summary.

Hand in your solutions to problems marked "HW" and "BONUS." Do not hand in problems marked "DO." Warning: the BONUS problems are underrated. PRINT YOUR NAME ON EVERY SHEET you submit. **Use LaTeX to typeset your solutions**. (You may draw diagrams by hand.) Hand in your solutions on paper, do not email. If you hand in solutions to CHALLENGE problems, do so on a **separate sheet**, clearly marked "CHALLENGE," and notify the instructor by email to make sure it won't be overlooked.

Carefully study the policy (stated on the website) on collaboration, internet use, and academic integrity. **State collaborations and sources both in your paper and in email to the instructor.**

Definitions, notation. As before $G = (V, E)$ denotes a graph with n vertices and m edges.

7.1 DO: The second quiz has been posted. Solve the problems (including the Bonus problems) on your own time before Thursday's class.

7.2 HW (7 points): Solve Problem 6 from the quiz: For every even n, construct a 1-factorization of K_n. Please accompany your solution with a picture for $n = 8$ that illustrates the general idea. (You may draw the picture by hand, no LaTeX required for the picture.)

7.3 DO: We say that G is a *minimal k-chromatic graph* if $\chi(G) = k$ and for every vertex v, we have $\chi(G \setminus v) = k - 1$. Prove: if G is a minimal k-chromatic graph then every vertex of G has degree $\geq k-1$. [Typo corrected 4-25]

7.4 HW (10 points): Construct the smallest triangle-free graph that is NOT 3-colorable. This graph has 11 vertices and it can be drawn in the plane to have a 5-fold symmetry (the picture won't change if you rotate it by $2\pi/5$). (The graph is not planar, the edges will cross in the picture.) You can draw the picture by hand; make your drawing nice. Number the vertices for reference. Prove that 3 colors do not suffice. Do not prove that it is the smallest.

7.5 CHALLENGE (10 points): Prove: for every k there is a triangle-free graph of chromatic number $\geq k$. DO NOT USE web sources.

7.∞ DO: More problems to follow, please check back later.