Honors Combinatorics CMSC-27410/Math-28400
Third Quiz. May 24, 2016
Instructor: László Babai

Name (print): ________________________________ **Major/Year** ____________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may **continue on the reverse.** This quiz contributes 5% to your course grade.

Warning: the BONUS problems are underrated; solve them only after you solved the ordinary problems. You are not expected to solve the bonus problems unless you seek graduate credit for this class.

1. (8 points) Find the number of solutions to the equation $x_1 + \cdots + x_k = n$ in positive **odd** integers $x_1, \ldots, x_n$. Your answer should be a simple closed-form expression in terms of k and n. (Case distinction permitted.) Prove your answer.

2. (30 points) Prove: For almost all graphs G with n vertices, $\alpha(G) \leq 1 + 2\log_2 n$. — Meaning of "almost all": Let p_n denote the probability that a random graph with n vertices satisfies the stated inequality. You need to prove that $p_n \to 1$ as $n \to \infty$. ("Random graphs" are generated by flipping a fair coin for each pair of vertices to decide adjacency.) (Hint: you need to reproduce a proof seen in class.) Do not worry about rounding. Write $k = 1 + 2\log_2 n$; don't expand the actual value of k until the very end of the proof.

3. (12 points) Prove the Oddtown Theorem: If $A_1, \ldots, A_m \subseteq [n]$ are sets such that (i) $(\forall i)(|A_i|$ is odd) and (ii) $(\forall i \neq j)(|A_i \cap A_j|$ is even) then $m \leq n$.

4. (BONUS 8 points) Fact (Esther Klein): Among any 5 points in the plane with no three on a line, there are 4 points that span a convex 4-gon. (Do not prove.)

   Use this fact, combined with Ramsey's Theorem, to prove the following (Erdős–Szekeres): For every k there exists n such that among any n points in the plane with no three on a line, there are k points that span a convex k-gon. — State the exact case of Ramsey's Theorem you are using.