Graph Isomorphism course, Spring 2017
Instructor: László Babai
Notes by Angela Wu and instructor
Thursday, May 25, 2017

18   Day 18, ThWk9

DO 18.1. Study those parts of the paper [GIQ] tha cover today's material:

- Section 3 ("Classical coherent configurations"), especially Sec. 3.4 ("Toward the analysis of combinatorial partitioning")

- Section 10 ("Split-or-Johnson"): get some idea of what is in each subsection, and study Sec. 10.6 ("Imprimitive case") and Sec. 10.8 ("UPCC case") in detail.

Let me know if you find errors.

DO 18.2 (Twin awareness lemma)**.** Let $\mathfrak{X} = (V; c)$ be CC (coherent configuration) where $c : V \times V \to \{\text{colors}\}$ is the coloring. Let i be a color and $R_i = c^{-1}(i)$ the corresponding constituent. Prove: the color $c(x, y)$ $(x \neq y \in V)$ determines whether or not x and y are twins with respect to the constituent R_i. (Prove this without reading the proof in the paper.)

HW 18.3. Let $\mathfrak{X} = (V; c)$ be a UPCC (uniprimitive coherent configuration). Let i be a dominant color, i.e., $\deg_i^+ \geq n/2$. Let $R_i = c^{-1}(i)$ be the corresponding constituent (the set of edges of color i). Then

1. $R_i = R_i^{-1}$ (the color-i constituent is an undirected graph)

2. the complement of the graph (V, R_i) has diameter 2.

DO 18.4. The symmetry defect of a nontrivial semiregular bipartite graph is $\geq 1/2$ in each part. (Here we are referring to the defect as defined for bipartite graphs, a separate quantity for each part. See paper.)