$\boxed{10-4-2022}$ p1

$A, B \subseteq \mathbb{Z}$

DEF $A+B = \{a+b \mid a \in A, b \in B\}$

$A = \{1, 3, 6\}$

$B = \{-1, 0, 1\}$

$A+B = \{0, 1, 2, \cancel{2}, 3, 4, 5, 6, 7\}$

$[n] = \{1, \dots, n\}$ $\quad n \geq 0 \quad [0] = \emptyset$

$[3] = \{1, 2, 3\}$

$[n] + [n] = \underline{[2n]} \setminus \{1\} = \{2, 3, \dots, 2n\}$

$\{1, 2\} + \{1, 2\} = \{2, 3, 4\}$

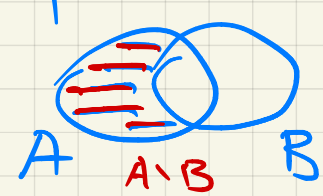
A, B sets

DEF $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$

set difference

$x \in A$

x \in A



P2

$A, B \subseteq \mathbb{Z}$
$|A| = k$
$|B| = \ell$

DEF $A+B = \{a+b \mid a \in A, b \in B\}$ sumset

$|A+B| \underset{?}{\leq} k \cdot \ell$ ($\boxed{HW}$ tight for all k, ℓ)

e.g. $k = 100$
$\ell = 1000$

$\boxed{HW}$ $|A+B| \geq$

Notation $|A| =$ # elements of A

$|A+B| \overset{*}{\geq} k + \ell - 1$
assuming $k, \ell \geq 1$

need: often $a + b = a' + b'$

$\boxed{A + \emptyset = \emptyset}$

$a' = a+1$
$b' = b-1$

$5 \longrightarrow 6 \longrightarrow 7 \longrightarrow 8$

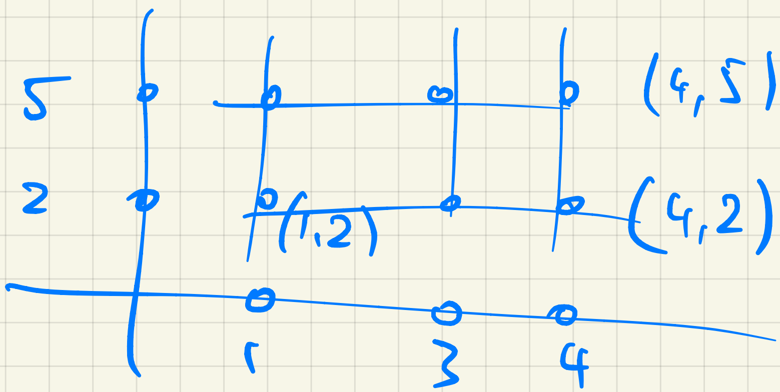
* is tight for all k, ℓ

$|[k] + [\ell]| = k + \ell - 1$

A, B sets

DEF $A \times B = \{(a,b) \mid a \in A, b \in B\}$

Cartesian product



illustration

$A = \{1, 3, 4\}$

$B = \{2, 5\}$

$$\boxed{A \times B = \{(1,2), (1,5), (3,2), (3,5), (4,2), (4,5)\}}$$

$|A \times B| = |A| \cdot |B|$ — Def of multiplication

if $A \cap B = \emptyset$

then $|A \cup B| = |A| + |B|$ Def of addition

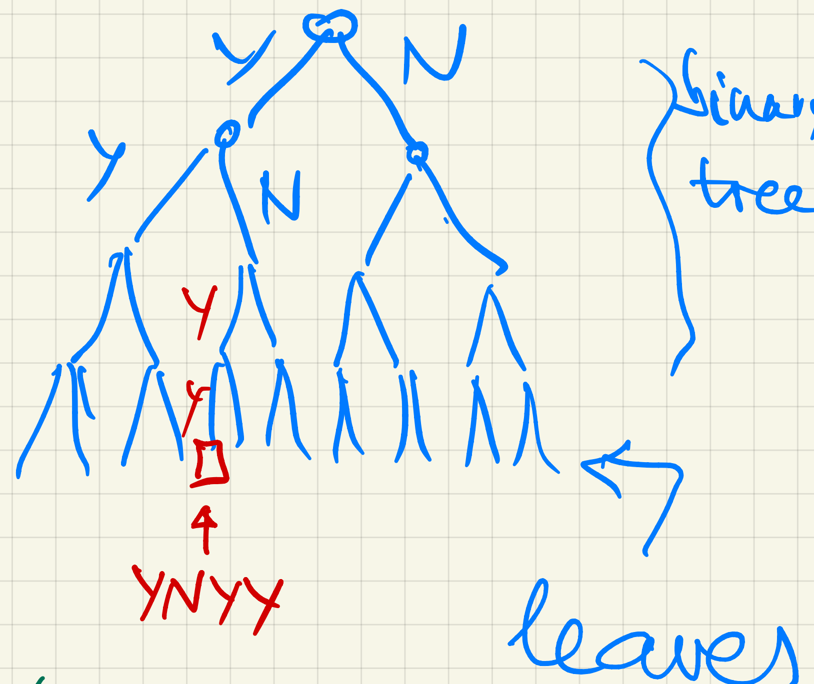
Set A

$\mathcal{P}(A)$ power set : $\mathcal{P}(A) = \{B \mid B \subseteq A\}$

set of all subsets of A

thm. If $|A| = n$ then $|\mathcal{P}(A)| = 2^n$

Jack	N
billy	Y
⋮	
	N
	N
	Y



case / subcase / sub-subcase / ...

$\Sigma = \{Y, N\}$ word of length 4: YNYY

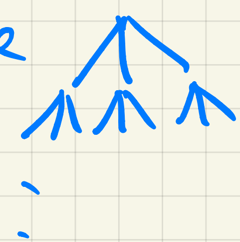
$|\Sigma| = k$
alphabet of size k

$\Sigma^n = \{$all word (strings) of length $n\}$

$|\Sigma^n| = ?$

strings of length n over Σ

example: $\Sigma = \{A, B, C\}$ $\longrightarrow$ ternary tree



$r_n = |\Sigma^n|$

$\boxed{r_n = 3 \cdot r_{n-1}}$

$\boxed{r_0 = 1}$

$\boxed{B}\underbrace{\boxed{\quad\quad\quad\quad}}_{n-1 \text{ missing letters}}$

P6

$$\boxed{\begin{aligned} r_0 &= 1 \\ r_n &= 3 \cdot r_{n-1} \end{aligned}}$$

$\leftarrow$ initial value

$\leftarrow$ recurrence

determine the sequence

sequence $r_0, r_1, r_2, \ldots, r_n$

$1, 3, 9, \ldots, 3^n$

then

If $|\Sigma| = k$ then

$$|\Sigma^n| = k^n$$

A, B sets $\quad |A| = k \quad |B| = \ell$

DEF Relation from A to B $\qquad A \to B$

$$R \subseteq A \times B$$

$A :=$ domain of R $\qquad \mathrm{dom}(R)$

$B :=$ codomain of R $\qquad \mathrm{codom}(R)$

Q # relations $A \to B$

set of relations : $\mathcal{P}(A \times B)$

$\therefore$ # relations$_{A \to B}$: $|\mathcal{P}(A \times B)| = 2^{|A \times B|} = 2^{k \cdot \ell}$

DEF Relation on A: $\quad A \to A$ relation

relation on A

$R \subseteq A \times A$

if $(x,y) \in R$ we write xRy

<u>example</u>: $A = \mathbb{Z}$

divisibility $R = \{(a,b) \mid a, b \in \mathbb{Z},\ a \mid b\}$

order $Ord = \{(a,b) \mid \text{—"—},\ a < b\}$

DEF R is <u>reflexive</u> if $(\forall a \in A)(aRa)$

$\boxed{\text{HW}}$ # " relations on A $|A| = n$

DEF R is <u>symmetric</u> if $(\forall a, b \in A)(aRb \Rightarrow bRa)$

$\boxed{\text{HW}}$ # —"— relations

refl. & symm. "