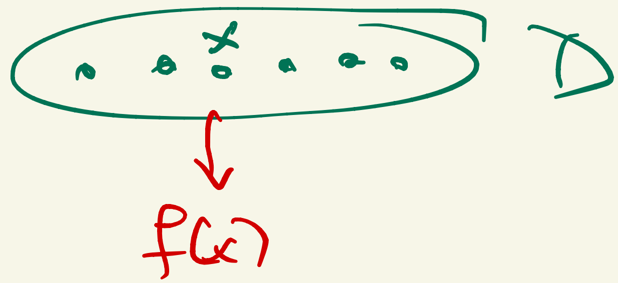
2022-10-13 | p1

Summation symbol

$$\sum_{i=1}^{n} a_i = a_1 + \cdots + a_n$$

$a_1, \ldots, a_n \in \mathbb{R}$

$$f : D \longrightarrow \mathbb{R}$$



$$\sum_{x \in D} f(x)$$

$$\sum_{i=1}^{n} a_i = \sum_{i \in [n]} a_i$$

$$\boxed{\sum_{x \in \emptyset} f(x) = 0}$$

LP2

(*) $$\sum_{x \in A \cup B} f(x) = \underbrace{\sum_{x \in A} f(x) + \sum_{x \in B} f(x)}$$

(+) true assuming $\underline{A \cap B = \emptyset}$

(*) $\overset{?}{\Longrightarrow}$ (+)

✓ (+) $\overset{?}{\Longrightarrow}$ (*)

(+) necessary for (*)

(+) sufficient for (*)

yet $\sum_{A} + \sum_{B} = \sum_{A \cup B}$ can be true

$$\sum_{x \in A \cap B} f(x) = 0$$

p3

HW

#terms in this sum

$$\underbrace{\sum_{1\le i\le j\le n} a_i a_j}$$

$=$

INPUT

$a_1 \ldots a_n$

$$= \sum_{(i,j)\in S} (a_i + a_j)$$

where

$$\underbrace{S = \{(i,j) \mid 1\le i\le j\le n\}}$$

HW

#terms

$$\sum_{1\le i\le j\le k\le n} a_i a_j a_k$$

LaTeX

\sum_{ }

Cont.

$$\prod_{i=1}^{n} a_i = a_1 \cdots a_n$$ product symb.

\prod_{i=1}^n

$$\prod_{1\le i\le j\le n} (a_i + a_j)$$

\prod_{1\le i\le j\le n} (a_i+a_j)

\le

$$n! = \prod_{i=1}^{n} i$$

$$\sum_{x\in A} 1 = |A|$$

$$\boxed{\sum_{B\subseteq A} 1 = 2^n}$$

$\boxed{\text{Hw}}$ $$\sum_{B\subseteq A} |B|$$

<u>A given</u> $|A|=n$

Univ: $\mathbb{Z}$

DEF $a|b$

We say that $a|b$ if $(\exists x)(ax=b)$

$\text{Div}(a) = \{\text{divisors of } \underline{a}\} = \{k : k|a\}$

$\text{Div}(6) = \{\pm 1, \pm 2, \pm 3, \pm 6\}$ $\quad |\text{Div}(6)| = 8$

$\text{Div}(5) = \{\pm 1, \pm 5\}$ $\quad |\text{Div}(5)| = 4$

DEF p is a prime number if $p \geq 1$ and $|\text{Div}(p)| = 4$

$\text{Div}(1) = \{\pm 1\}$

$\text{Div}(0) = \mathbb{Z}$

$\text{Div}(-a) = \text{Div}(a)$

$$\mathrm{Div}(a) = \mathrm{Div}(b) \iff a = \pm b$$
$$\iff |a| = |b|$$

DEF prime factorization of $n \geq 1$

is a sequence $p_1 \dots p_k$ of prime numbers

s.t. $\prod_{i=1}^{k} p_i = n$

Ex: $12 = 2 \cdot 3 \cdot 2$

$2 \cdot 2 \cdot 3$

$3 \cdot 2 \cdot 2$

FUNDAMENTAL THM OF ARITHM:

Uniqueness of prime factorization

up to ordering of the factors

Existence of prime factorization: $\forall n \geq 1$ has a prime factorization

HW/DO by induction

Factorization of $n=1$

$$\boxed{1 = \prod_{i \in \emptyset} p_i}$$

$$a \cdot b = \prod p_i \cdot \prod q_j$$

$$a = \prod_{i=1}^{k} p_i \qquad b = \prod_{j=1}^{\ell} q_j$$

if $a=1$

$ab = b = \prod q_j$

$\prod p_i = 1$

proof of uniqueness appears in "Elements" by EUCLID ($\approx$ 300 B.C.E.)

EUCLID's Lemma

If p is a prime and $p \mid ab$ then $p \mid a$ or $p \mid b$

♥ of pf of Fund. Thm. of Arithm

False if $p=6$: ex: $a=2$, $b=3$

DEF n is **composite** if $n \geq 2$ and n is not prime

HW EUCLID's lemma $\Longrightarrow$ FTA

by induction

naive def of GCD : $\underline{\max}$ Div(a,b)

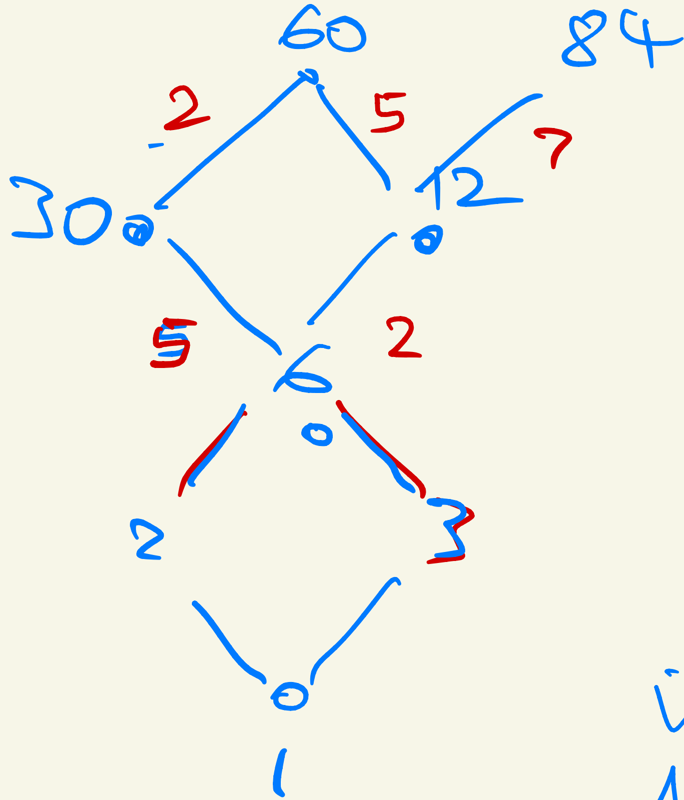
$\mathrm{Div}(a,b) = \underline{\mathrm{Div}(a)} \cap \underline{\mathrm{Div}(b)}$

Issue $\gcd(a,a) = |a|$ $\gcd(0,0)=0$

$\boxed{\text{is } a=0 \text{ an exception?}}$ $\mathrm{Div}(0)=\mathbb{Z}$ $\mathrm{Div}(0,0)=\mathbb{Z}$

$\mathrm{Div}(-6) = \{\pm1, \pm2, \pm3, \pm6\}$ $\gcd(-6,-6)=6 \leftarrow$

divisibility hierarchy



$2\cdot 3\cdot 5 \quad 2\cdot 3\cdot 13$

$\gcd(30,78) = 2\cdot 3 = 6$

$\mathrm{Div}(30,78) = \mathrm{Div}(6)$

in this hierarchy,

1 is at the bottom $(\forall x)(1 \mid x)$

and 0 is at the top $(\forall x)(x \mid 0)$

raises question of existence

DEF by wish list: $|d| = \gcd(a,b)$ if

(1) $d \in \mathrm{Div}(a,b)$ — d is a common divisor

(2) $(\forall e)(e \in \mathrm{Div}(a,b) \Rightarrow e \mid d)$ — d is divisible by all common divisors