DEF $a_i \in \mathbb{R}, \mathbb{C}$

$S = (a_0, a_1, \dots)$ is <u>PERIODIC</u> if $\boxed{\exists d \in \mathbb{N}}$ (pos. integer)

s.t. $(\forall i)(a_{i+d} = a_i)$ with period $\underline{\underline{d}}$

DO If r, s are periods of S then $\gcd(r,s)$ is also a period

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<u>Example</u> $1,2,5,1,2,5,1,2,5$ periodic with period 3

$$\frac{1}{7} = 0.142857142857142857\dots$$

$$\frac{1}{3} = 0.3333\dots$$

<u>Reward</u> p prime $\neq 2,5$ then the sequence of decimal digits of $\frac{1}{p}$ are periodic w period $\underline{\underline{p-1}}$

DO shortest period | any period

$S \subseteq \mathbb{Z}$

d is a gr. c. d. of S if

(a) $(\forall a)(a \in S \Rightarrow d \mid a)$ common divisor

(b) if $(\forall a)(a \in S \Rightarrow e \mid a) \Longrightarrow e \mid d$

DO $\forall S$ $\exists$ gr. c. d.

DO gr. c. d. is unique up to sign $\pm$

NOTATION $\gcd(S) = |d|$ for either of the two gr. c. d's d

DO shortest period $=$ gcd(all periods)

(p3

DEF $a_0, a_1, \ldots$ is eventually periodic

if $(\exists d)(\exists n_0)(\forall i)(i \geq n_0 \Rightarrow a_{i+d} = a_i)$
$d \in \mathbb{N}$

Reward The sequence of decimal digits of $x \in (0,1)$ is eventually periodic $\Longleftrightarrow$ x is rational

LIMIT of a sequence

$a_0, a_1, a_2, \ldots, L \in \mathbb{R}$ or $\mathbb{C}$

DEF We say that

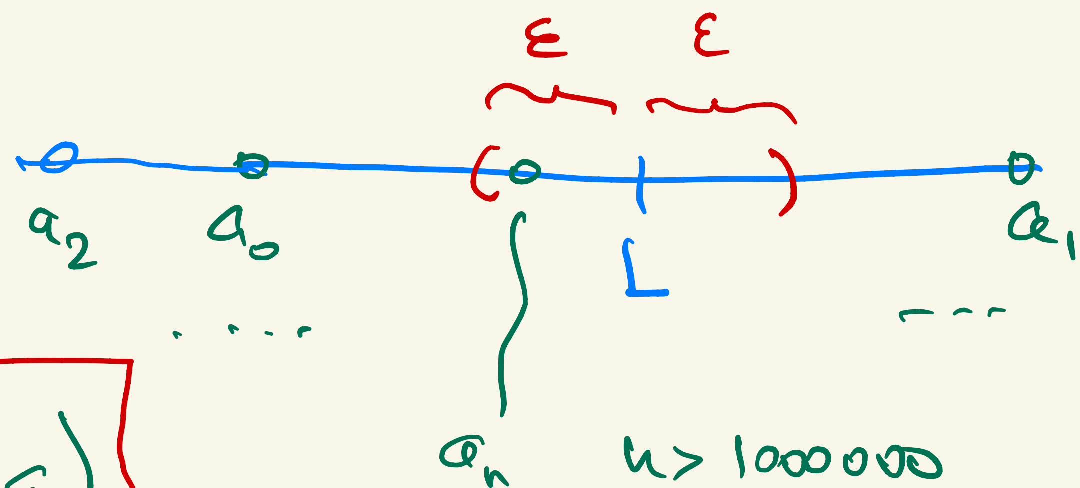
$$\lim_{n\to\infty} a_n = L$$

"the limit of a_n as n goes to infinity is L"
"a_n converges to L"
"a_n goes to L"
"a_n approaches L"

if

$$\boxed{(\forall \varepsilon > 0)(\exists n_0)(\forall n > n_0)(|a_n - L| < \varepsilon)}$$

n_0: threshold



alternating quantifiers: game

formula true $\iff$ existential player has winning strategy

HW Define

$$\lim_{n\to\infty} a_n = \infty$$

Examples of a sequence with no limit

$a_n = (-1)^n$ 1, −1, 1, −1, ... (*)

1, −2, 3, −4, 5, ... (**)

DO If $\lim_{n\to\infty} a_n = L$

then same is true for every subsequence

DO use this to prove (*), (**) have no limit

Rates of growth

a_n, b_n sequences

DEF We say that (a_n) ASYMPTOTICALLY EQUAL to (b_n) if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 1$$

NOTATION $a_n \sim b_n$

LaTeX \sim

Examples

$$a_n = 3n^3 - 100n^2 + 7$$

$$b_n = 3n^3$$

$$\frac{a_n}{b_n} = 1 - \frac{100}{3} \cdot \frac{1}{n} + 7 \cdot \frac{1}{n^2} \longrightarrow 1 \quad \text{so } a_n \sim b_n$$

T/F: for all sequences

$$a_n \overset{?}{\sim} a_n$$

FALSE counterexample: $\forall n \quad a_n = 0$

b_n needs to be EVENTUALLY NONZERO

<u>DO</u> $\neg$(eventually nonzero) = "plain English"