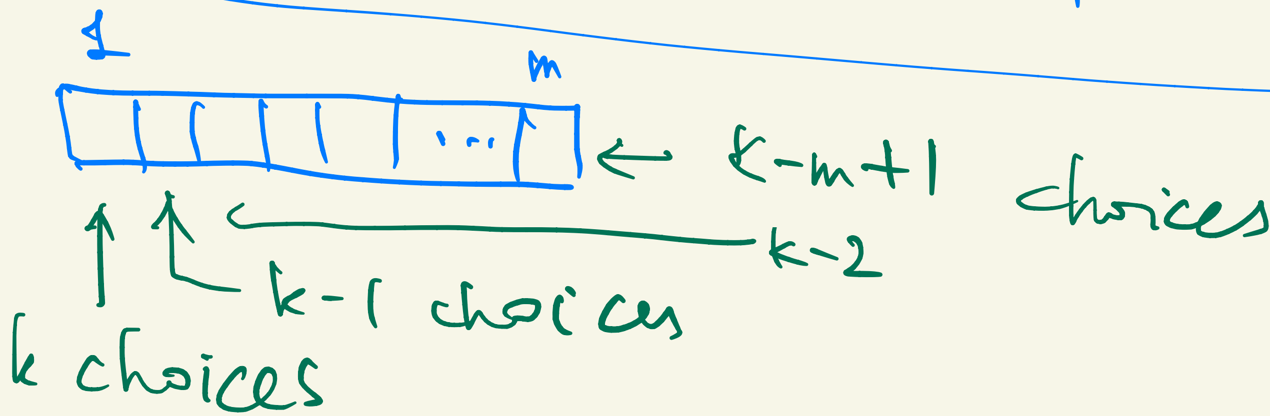
$|A| = m \quad |B| = k$

2.11 counting injections $f : A \to B$

s.t. $(\forall x, y \in A)(f(x) = f(y) \Rightarrow x = y)$



total $\underbrace{k(k-1)\cdots(k-m+1)}_{\text{product of } \underline{m} \text{ terms}} = \prod_{j=0}^{m-1}(k-j)$

$$= \frac{k!}{(k-m)!}$$

$y^3 + x(x-1)$

closed-form expression

$x^k = \underbrace{x \cdot \cdots \cdot x}_{k \text{ times}}$

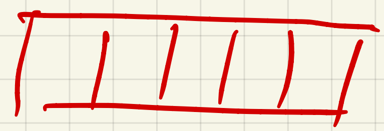
x^k "closed form" by def

$n! = \prod_{j=1}^{n} j$ — " —

n-factorial

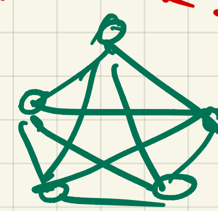
a, b, c, d

(b, d, a, c)



\# choices $k \quad k-1 \quad \cdots \quad 1$

total $k(k-1)\cdots 1 = k!$



$n, k \geq 0$

$\binom{n}{k}$:= # k-subsets of an n-set

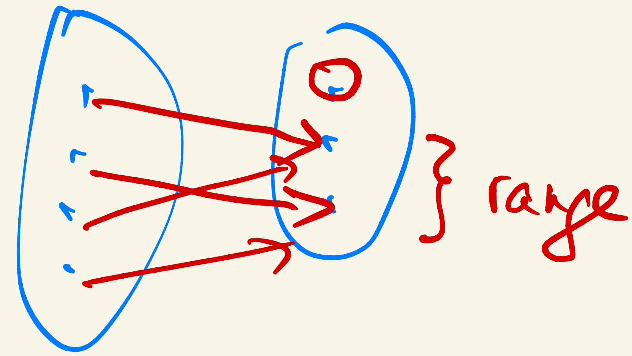
$= \frac{n(n-1)\cdots(n-k+1)}{k!}$

example $\binom{5}{2} = 10$

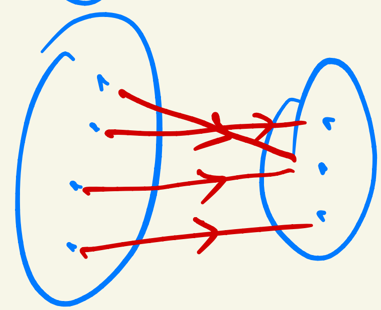
$= \frac{5 \cdot 4}{2 \cdot 1}$

fctn $f : A \to B$ is a <u>surjection</u>

if <u>$\mathrm{range}(f) = B$</u>



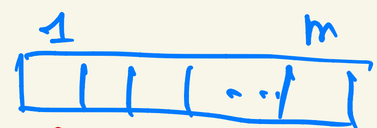
DEF $\mathrm{range}(f) = \{ f(x) \mid x \in A \}$



let $|A| = m$, $|B| = 2$

Set of fctns $f : A \to B$

denoted B^A

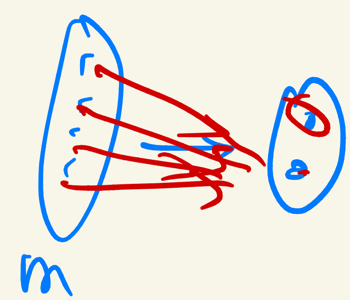


$2 \cdot 2 \cdot 2 \cdots 2$

choices

$|B^A| = 2^m$

<u>not</u> surjection: 2 (const. fctns)



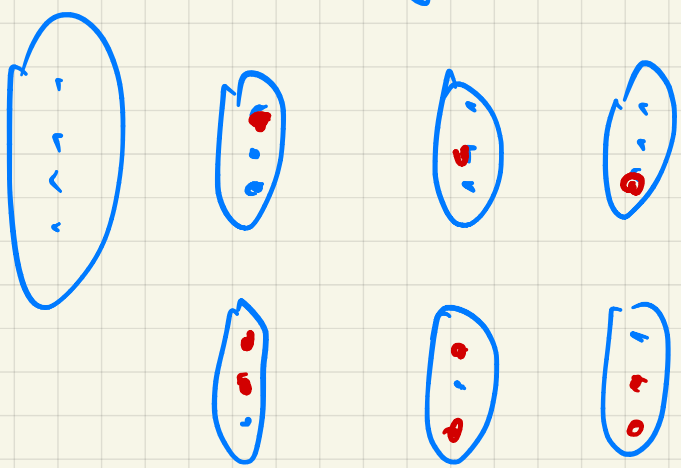
surjections: $\underline{\underline{2^m - 2}}$

2.23 XC

\# surj $A \to B$ $\qquad |A| = m$, $|B| = 3$

\# functions $A \to B$ is 3^n

how many of these have $|\text{Range}| = 1$ $\qquad \underline{\underline{3}}$.



} R := range

$|\text{Range}| = 2$

total $3 \cdot (2^m - 2) = \underline{\underline{3 \cdot 2^m - 6}}$.

\# $A \to R$ surjections $2^m - 1$

grand total: $3 \cdot 2^m - 3$

\# SURJECTIONS: $3^m - 3 \cdot 2^m + 3 \cdot 1^m$

Poker hand: set of 5 out of $\underbrace{\text{52 cards}}_{\text{standard deck}}$

standard deck (STUDY!)

$|\Omega| = \binom{52}{5}$ ← # poker hands

full house: 3 of a kind + 2 of a kind

ex. 3 Kings, 2 9s

$52 = 13 \times 4$

♥ ♦
♠ ♣

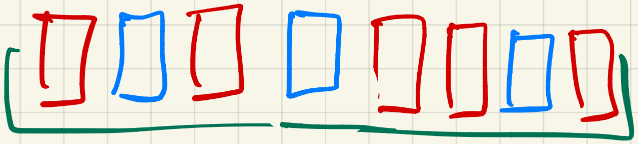
full house hands

$$13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2} = 13 \cdot 12 \cdot 4 \cdot 6$$

choices of first kind

2nd kind

NAIVE PROBABILITY



$$P(\text{blue}) = \frac{3}{8}$$

$$P = \frac{\#\text{ good outcomes}}{\#\text{ outcomes}}$$

$$P(\text{full house}) = \frac{13 \cdot 12 \cdot 4 \cdot 6}{\binom{52}{5}}$$

---

n coin flips : outcome $\boxed{H}\boxed{T}THTHHHH$

$$|\Omega| = 2^n$$

sample space: set of outcomes of the experiment n coin flips ← experiment

$$\underbrace{P(k \text{ are Heads})}_{\text{event}} = \frac{\binom{n}{k}}{2^n}$$

| H | T | H | H | . | . | . |

H's is $\underline{\underline{k}}$

outcomes without consecutive heads

HTTHTHTTT

~~HTHHHTTH~~

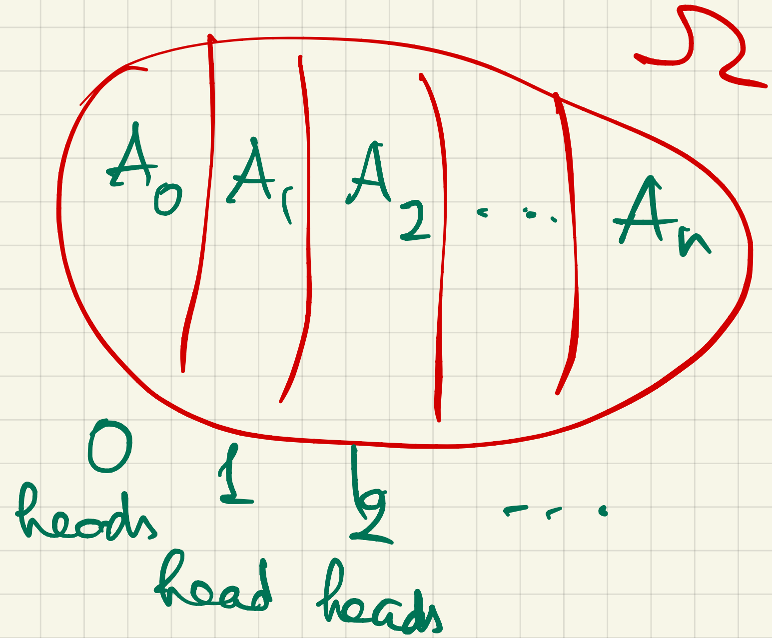
HW $\quad \binom{n}{k} \geq \left(\frac{n}{k}\right)^k$ for $1 \leq k \leq n$

XC $\quad \binom{n}{k} < \left(\frac{en}{k}\right)^k$

$e = 2.71\ldots$

8)

$$\sum_{k=0}^{n} \frac{\binom{n}{k}}{2^n} = 1$$



$\binom{n}{0} = 1$

$\binom{n}{n} = 1$

$$\frac{\binom{n}{0}}{2^n} + \frac{\binom{n}{1}}{2^n} + \dots + \frac{\binom{n}{n}}{2^n}$$

$A_i \subseteq \Omega$, $A_i \cap A_j = \emptyset$,

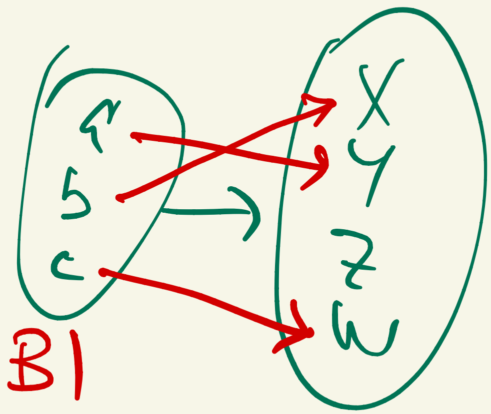
$\bigcup_{i=0}^{n} A_i = \Omega$

partition of Ω

$\therefore \sum |A_i| = |\Omega|$

$\exists\ A \to B$ injection $\iff |A| \le |B|$

PIGEON HOLE PRINCIPLE



$\exists\ A \to B$ surjection $\iff |A| \ge |B|$

HW $B(n)$ n^{th} Bell number:
number of partitions of an n-set

Prove: $B(n) \le n!$ give an injective proof

DO $B(0), B(1), B(3)$