DEF a_n sequence of reals, $L \in \mathbb{R}$

We say that $\lim_{n\to\infty} a_n = L$ $[\, a_n \to L \,]$

if

$$(\forall \varepsilon > 0)(\exists n_0)(\forall n)(n > n_0 \Rightarrow |a_n - L| < \varepsilon)$$

DEF $a_n \to \infty$ $\qquad \lim_{n\to\infty} a_n = \infty$

$$(\forall K \quad)(\exists n_0)(\forall n)(n > n_0 \Longrightarrow a_n > K)$$

eventually $a_n > K$

for all sufficiently large n, we have $a_n > K$

(a_n) is eventually nonzero if

sequence

$$(\exists n_0)(\forall n)(n>n_0 \Rightarrow a_n \neq 0)$$

READ ASY

$\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{N}_0 = \mathbb{N} \cup \{0\}$

$(a_n \mid n \in \mathbb{N}_0)$ → $a_0, a_1, a_2, \ldots$

$a_3, a_7, a_8, a_{20}, \ldots$

$I \subseteq \mathbb{N}_0$ infinite

$I = (3, 7, 8, 20, \ldots)$

$(a_n \mid n \in I)$

DEF $\lim\limits_{n \in I} a_n = L$ if $(\forall \varepsilon>0)(\exists n_0)(n>n_0 \wedge n \in I \Rightarrow |a_n - L| < \varepsilon)$

DEF Let $\underbrace{J}_{\text{infinite}} \subseteq I \subseteq \mathbb{N}_0$

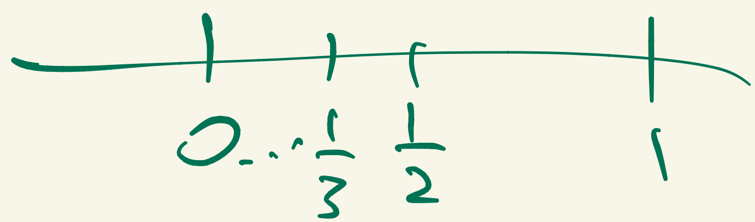
Then we say that $(a_n \mid n \in J)$ is a <u>subsequence</u> of $(a_n \mid n \in I)$

[TO] If $\lim\limits_{n \in I} a_n = L$ then $\lim\limits_{n \in J} a_n = L$

<u>Example</u> $\lim\limits_{n \to \infty} \frac{1}{n} = 0$

$\lim\limits_{n \in \{2^k \mid k \in \mathbb{N}_0\}} \frac{1}{n} =$

$= \lim\limits_{k \to \infty} \frac{1}{2^k} = 0$

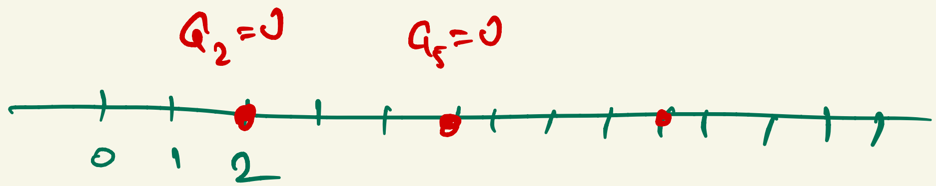


a_n is event. nonzero $(\exists n_0)(\forall n)(n>n_0 \Rightarrow a_n \neq 0)$

"a_n is NOT eventually nonzero"

$$(\forall n_0)(\exists n)(n>n_0 \wedge a_n = 0)$$

"infinitely often $a_n = 0$"



$$\neg(A \to B) \Leftrightarrow A \wedge (\neg B)$$

find sequence (a_n) that is neither event. zero
nor " nonzero

$0, 1, 0, 1, 0, 1, \ldots$

HW If $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} a_n = M$

then $L = M$

"every sequence has at most one limit"

a_n 0, 1, 0, 1, ... has no limit b/c

a_{2k} : 0, 0, 0, ... $\longrightarrow$ 0

a_{2k+1} : 1, 1, 1, ... $\longrightarrow$ 1