PROBLEM SESSION | 2024-01-05 pm | 1

(1) Prove: $\gcd(5a+4, 7a+3) = \begin{cases} 1 \\ 13 \end{cases}$ $\gcd(55b+2, 34b-3) = \begin{cases} 1 \\ 233 \end{cases}$

(2) Find a_n, b_n s.t. $a_n \sim b_n$ but $a_n^n \neq O(b_n^n)$

$n! \sim \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$

(3) $\sqrt{n^2+1} - n \sim a \cdot n^b$ $a, b = ?$

(4) $(\forall n)\left(n! > \left(\frac{n}{e}\right)^n\right)$ ← Can we use Stirling's formula?

(5a) p prime, $x^2 \equiv 1 \pmod p \implies x \equiv \pm 1 \bmod p$

(5b) $\forall p \neq q$ odd primes $\implies \left(x^2 \equiv 1 \bmod (pq) \not\Rightarrow x \equiv \pm 1 \bmod (pq)\right)$

(6) $a_n^2 \sim b_n^2 \overset{?}{\not\Rightarrow} a_n \sim b_n$

(7) $\ln\left(1+\frac{1}{n}\right) \sim \frac{1}{n}$ prove

(8) $\sin\left(\frac{1}{n}\right) \sim \frac{1}{n}$