HONORS ALGORITHMS

2024-01-22

graphs : undirected

$G = (V, E)$ — V: set of vertices, E: set of edges

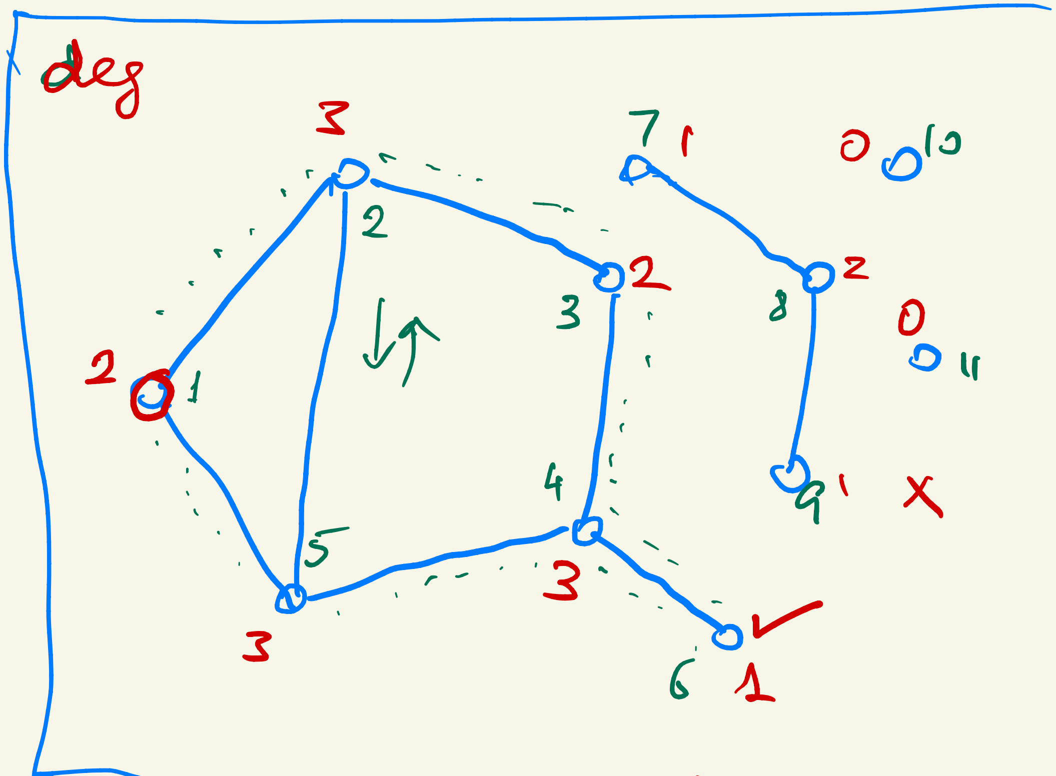
$V = [11]$

$E = \{\{1,2\}, \{2,5\}, \{2,3\}, \ldots, \{7,8\}, \{8,9\}\}$

$n = |V|$

$m = |E|$

$0 \le m \le \binom{n}{2} = \frac{n(n-1)}{2}$

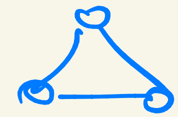


largest clique : K_3

If $m = \binom{n}{2}$ then G : complete graph

clique

K_n : clique on n vertices

K_3 

"triangle"

If $\{i,j\} \in E$ we say i, j are adjacent : $i \sim j$

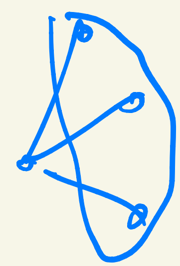
adjacency relation is

$i \sim_G j$

(1) irreflexive : $(\forall v \in V)(v \not\sim v)$

(2) symmetric : $v \sim w \implies w \sim v$

$i \sim j$ i, j neighbors

i  $N_G(i)$

degree $\deg(v) = |N_G(v)|$

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v isolated if $\deg(v) = 0$

$H \subseteq G$ if $V(H) \subseteq V(G)$

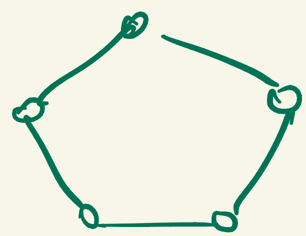
subgraph $\qquad$ $E(H) \subseteq E(G)$

G is triangle-free if $K_3 \nsubseteq G$

REWARD If G is Δ-free then $m \le \left\lfloor \frac{n^2}{4} \right\rfloor$

Cycle of length $n \geq 3$

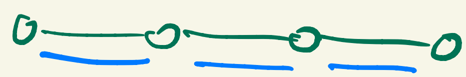
C_n



C_5

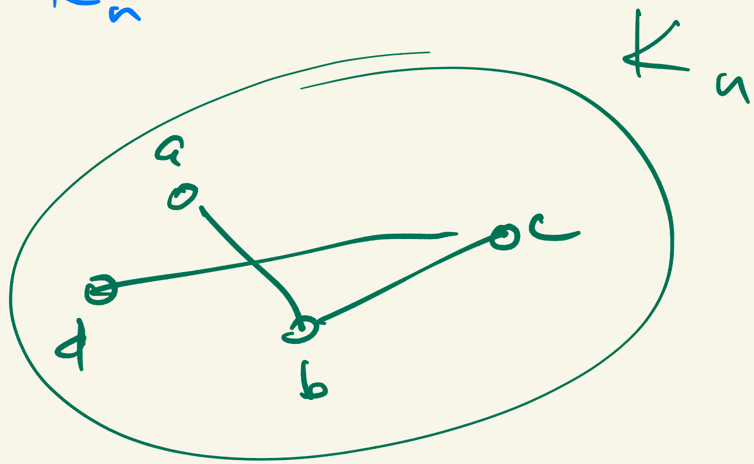
Path of length $n-1$

P_n ← n vertices



P_4

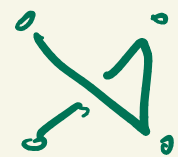
\# P_4 subgraphs of K_n



$$\frac{n\cdot(n-1)\cdot(n-2)\cdot(n-3)}{2}$$

\# choices for $\underline{a}$

$$= \binom{n}{4}\cdot 12$$



$$\frac{4!}{2} = 12$$

$\underline{w}$ is accessible from $\underline{v}$ if $\underline{\exists\ v \cdots w \text{ path}}$

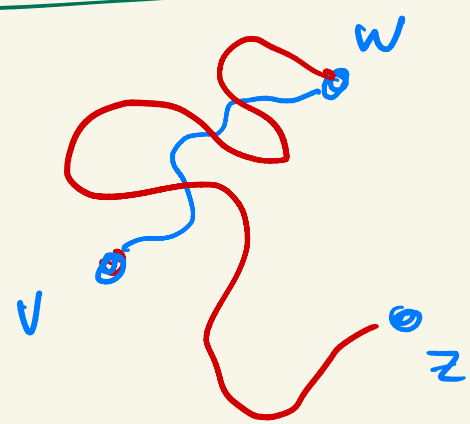
is v accessible from v? YES by P_1: length 0

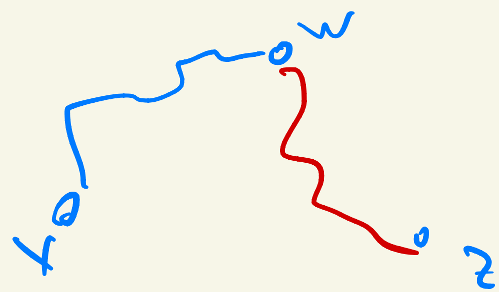
The accessibility relation is an <u>equivalence relation</u>

(1) reflexive $(\forall v \in V)(v - \cdots - v)$

(2) symmetric: $v - \cdots - w \Rightarrow v - \cdots - w$

(3) transitive: $v - \cdots - w \wedge w - \cdots - z \Rightarrow v - \cdots - z$

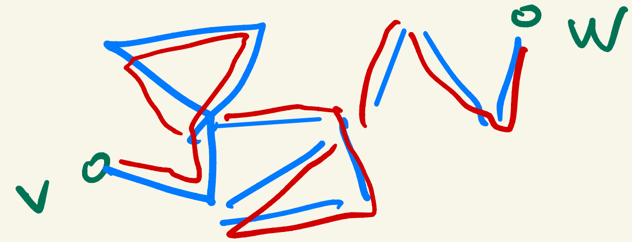




$v - \cdots - w$ walk of length k:

$v = v_0 - v_1 - \cdots - v_k = w$

from v to w
(has direction)



DO If $\exists\, v - \cdots - w$ <u>walk</u> then $\exists\, v - \cdots - w$ <u>path</u>

<u>EX</u>: write a pseudocode to reduce $v - - w$ walk to a $v - - w$ path

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<u>READ</u> Equivalence relations: Fall 2023 course note "Intro to math reasoning"

$\frac{5}{10}$ $\frac{3}{6}$

DEF connected components of graph: equivalence classes of the accessibility relation

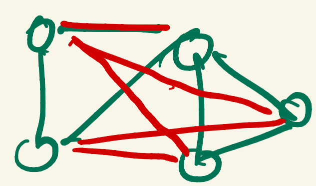
DEF G is connected if it has only one connected comp.

DO G is conn. $\iff (\forall v, w)(w$ is acc. from $v)$

$\overline{G} = (V, \overline{E})$ complement of G

$$(\forall v, w)(v \neq w \Rightarrow [v \sim_{\overline{G}} w \iff v \not\sim_G w])$$

Obs $m_G + m_{\overline{G}} = \binom{n}{2}$



$G \to H$

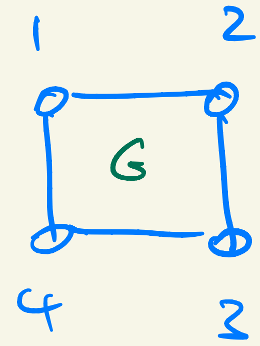
DEF **Isomorphism**

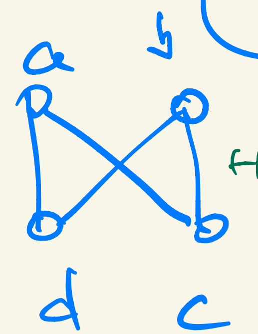
is a bijection

$$f : V(G) \to V(H)$$

s.t.

$$(\forall v, w \in V(G))\,\big(v \sim_G w \iff f(v) \sim_H f(w)\big)$$





$1 \to a = f(1)$ $\quad 1 \sim_G 2$

$2 \to c = f(2)$ $\quad a \sim_H c$

$3 \to b$

$4 \to d$

f bijection

$$f : V(G) \to V(H)$$

DEF $G \cong H$ $\quad$ G is isomorphic to H

if $\exists\, G \to H$ isomorphism

#bijections: $n!$

9

$$\text{dist}(v, w) = \text{length of shortest path } v - \cdots - w$$

$$= \infty \text{ if } \nexists\, v \cdots w \text{ path}$$