HONORS ALGORITHMS

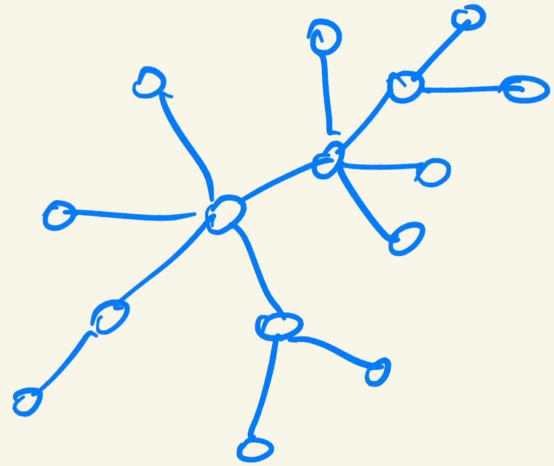
2024-01-26

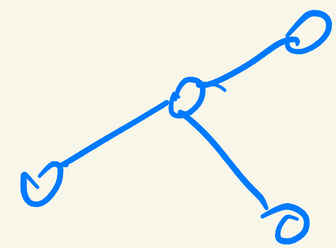
free tree: connected, cycle-free graph

DO G is a tree $\iff$

$(\forall u,v)(\underset{\text{unique}}{\exists!}\ \overset{u-v}{\text{path}})$

DO tree: $\underset{|E|}{m} = \underset{|V|}{n} - 1$





$n=4$

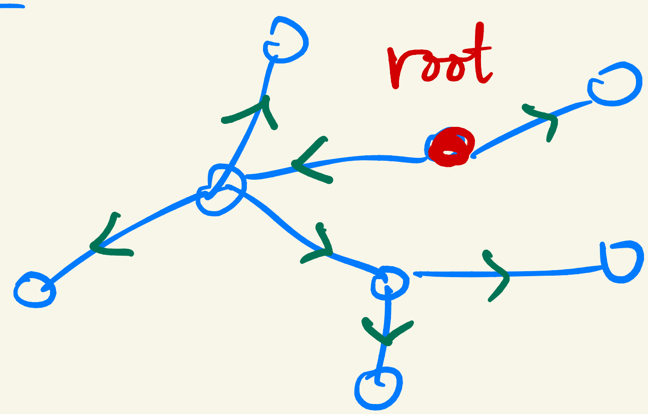
$m=3$

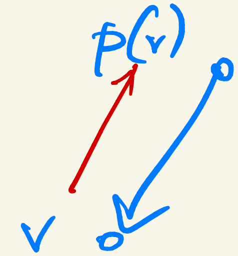
rooted tree

(T, v)

T - tree

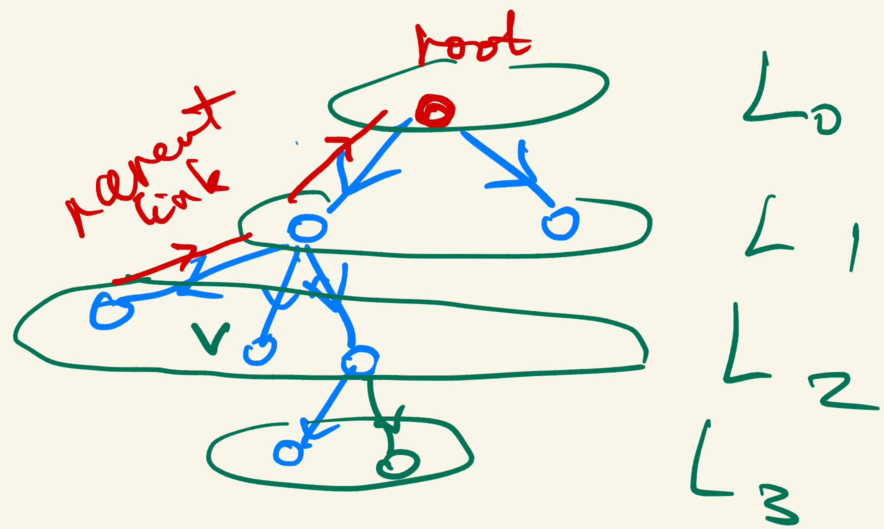
$v \in V(T)$





"CS" tree: rooted tree

edges oriented away from root



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what characterizes vertices in $L_k = \{v \mid dist(root, v) = k\}$ layers

G digraph, $s \in V(G)$ "source"

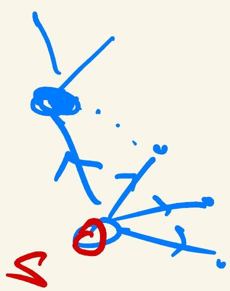
Question: Which vertices are accessible from s

What is their directed distance from s

How do I find shortest $s \to \cdots \to v$ paths

Linear time in unit cost model $V = [1..n]$

BREADTH FIRST SEACH BFS



status:

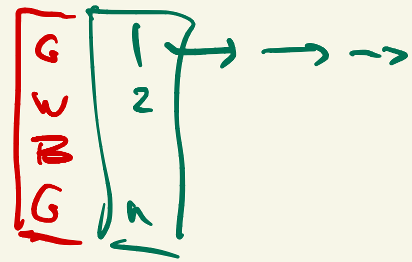
WHITE : not discovered yet

GRAY: discov. not finished yet

BLACK: finished

FIFO list

BFS (G, s)   G digraph



Initilize **for** $v \in V$

status(v) = WHITE

parent (v) = NIL

dist (v) = ∞

Q = $\emptyset$ empty FIFO queue

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status (s) := GRAY

enqueue(Q, s)   adding s to Q

dist := 0

parent (s) := S

---

while $Q \neq \emptyset$

u ← dequeue(Q)   u : oldest vertex in Q

status(u) := GRAY

for $w \in$ Adj [u]

if status (w) = WHITE

status(w) := GRAY   ← add w to Q

enqueue(w)

parent(w) := u

endfor

while $Q \neq \emptyset$

u ← dequeue(Q) — u : oldest vertex in Q

for $w \in Adj[u]$

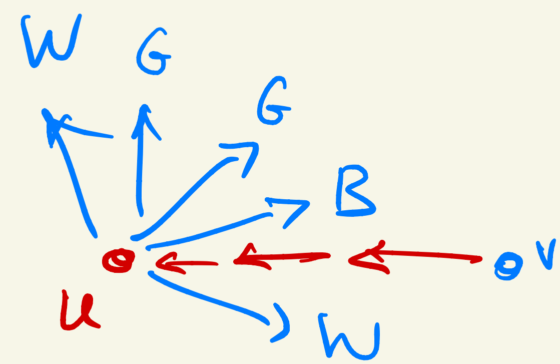
if status(w) = WHITE

status(w) := GRAY

enqueue(w) ← add w to Q

parent(w) := u

dist(w) := 1 + dist(u)



end for

status(u) := BLACK

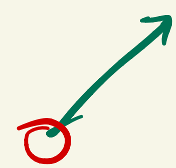
end while

Return parent dist status

array

LINEAR TIME

examines every edge at most once

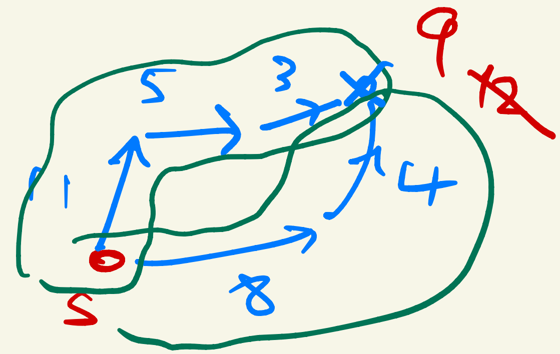


DO parent links define a tree rooted at s

the unique s → ⋯ → v path shortest in G

(G, s, w) $\quad w : E \to \mathbb{R}$

weighted edges



DIJKSTRA'S ALGORITHM

PRIORITY QUEUE

EXTRACT-MIN : finds/removes item with smallest key

INSERT (key) $\qquad$ ↑ real

UPDATE (item)

$s \in V$ source

Q: priority queue

for $v \in V$ $\quad Q = \emptyset$

status(v) = WHITE

$p(v)$ = NIL

cost(v) = ∞

---

status(s) := GRAY

INSERT(Q, s) $\quad$ cost[s] = 0

$p(s)$:= s

---

while $Q \neq \emptyset$

$u \leftarrow$ EXTRACT-MIN(Q)

for $v \in Adj[u]$

if status(v) = WHITE

status(v) := GRAY

RELAX(u, v) $\leftarrow$ updates parent ~~cost~~

INSERT(Q, v)

elseif status(v) = GRAY

RELAX(u, v)

endfor

status(u) := BLACK

endwhile