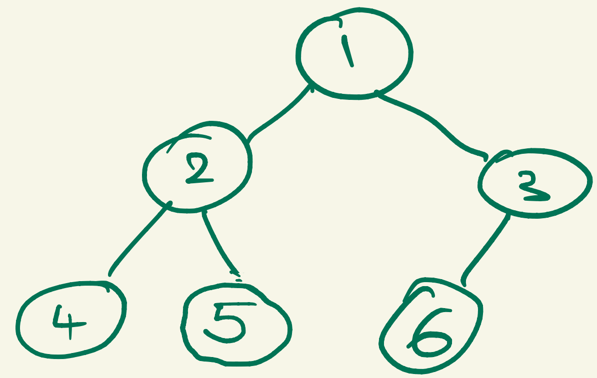
HONORS ALGORITHMS | 2024-01-31 | L1

REVIEWED HEAP IMPLEMENTATION OF PRIORITY QUEUE
from last 3 slides of previous class

ARRAY IMPLEMENTATION OF BALANCED BINARY TREE



$$\text{parent}(v) = \left\lfloor \frac{v}{2} \right\rfloor$$

$$\text{left.child}(v) = 2v$$

$$\text{right.child}(v) = 2v+1$$

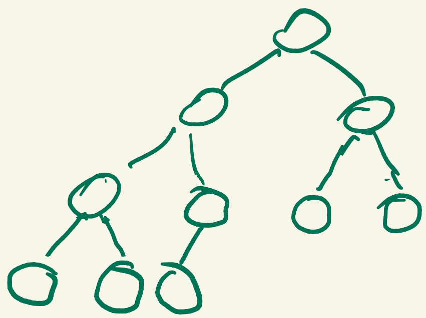
---

n INSERTS $\qquad n \cdot \log_2 n$

$$\underline{\log 1 + \dots + \log n} = \log(n!) \sim n \cdot \log n$$

Claim MAKE-HEAP

input: list of n data $\qquad O(n)$ comparisons



fill tree bottom-up

depth of tree : d

need to bubble-down each added node

node on level $d-i$ requires

$\leq 2i$ comparisons

nodes on level $d-i$: $\leq 2^{d-i}$

total # comparisons $\leq \sum_{i=0}^{d} 2^{d-i} \cdot 2i < 2^d \underbrace{\sum_{i=1}^{\infty} \frac{i}{2^{i-1}}}_{4} = 4 \cdot 2^d < 4n$

lower bound on # comparisons
made by Dijkstra
under "best" implementation of PRIORITY QUEUE

"worst" input

$\forall$ impl. $\exists$ input $= \Omega(n \cdot \log n + m)$

cost $\geq m$

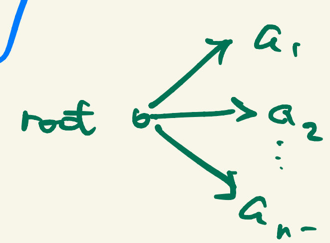
Claim cost $\geq \log(n!) \sim n \log n$

$2 \cdot \text{cost} \geq m + \log(n!)$

$\boxed{m \geq n-1}$

$m \leq n^2$

First: $m = n-1$



DIJKSTRA sorts these data

FIBONACCI HEAP

FREDMAN - TARJAN mid 1980s

INSERT: $O(1)$ $\times n$

EXTR-MIN: $O(\log n)$ $\times n$

DECREASE-KEY: $O(1)^*$ $\times m$

[INCR-KEY $O(\log n)$]

$$O(n \log n + m)$$

100111 +1

...1000

amortized cost $O(1)$

of INCREMENTING 0...n

$G = (V, E)$ undirected

Subgraph $H = (W, F)$

$H \subseteq G$ if $W \subseteq V$, $F \subseteq E$

Spanning subgraph $W = V$, $F \subseteq E$

Spanning tree H is a tree

$\boxed{\text{DO}}$ G has a sp. tree $\Longleftrightarrow$ G is connected