

PROBLEM 2024.D2.09

SESSION

1

12.66 sorting heap-organized data

als. A Assume k comparisons suffice if input is heap-organized

takes $\approx n \log_2 n$

$(\forall \varepsilon > 0)(\forall n \geq n_0)$ it takes $> n \log_2 (1-\varepsilon)$

B sorts:

① HEAPIFY ② A

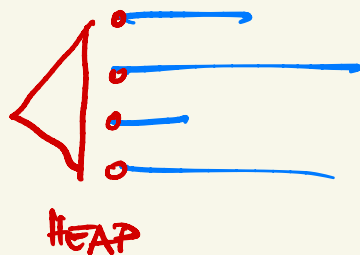
$$Cn + k \geq O(n) + k \approx n \log_2 n \geq (1-\varepsilon)n \log_2 n$$

$$k \geq (1-\varepsilon)n \log_2 n - Cn \sim (1-\varepsilon)n \log_2 n$$

12.59

2

k-way merge

merge in $O(n \log k)$ k sorted list
total length n $O(k)$ HEAPIFY the k min elements, remove them from each list

2 log k
 EXTRACT-MIN if MIN comes from list i
 INSERT in list i by INCREASE-KEY(root)

total $2n \log k$

bubble-down

2nd sol'n: D & C: $\text{MERGE}(\text{MERGE}_{\frac{k}{2}}, \text{MERGE}_{\frac{k}{2}})$

$$T(n) \leq T(l) + T(n-l) + n - 1$$

$$T(n, k) \leq T(l, k/2) + T(n-l, k/2) + n - 1$$

2nd sol'n: D&C : $\text{MERGE}(\text{MERGE}^{\frac{k}{2}}, \text{MERGE}^{\frac{k}{2}})$

$$T(n) \leq T(l) + T(n-l) + n-1$$

$$T(n, k) \leq T(l, k/2) + T(n-l, k/2) + n-1$$

$$g(n, k) = C n \log k$$

$$C n \log k \stackrel{?}{\geq} C l \log\left(\frac{k}{2}\right) + C(n-l) \log\left(\frac{k}{2}\right) + n-1$$

$$C n \log k \stackrel{?}{\geq} C l (\log k - 1) + C(n-l) (\log k - 1) + n-1$$

$$0 \geq -Cn + n-1$$

$$\underline{\underline{C=1}}$$

$n:=k$ $T(k, k) \sim k \log k$

12.62

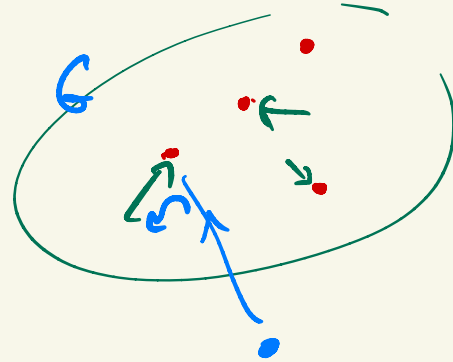
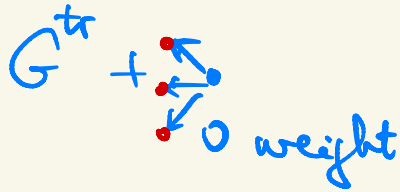
HEAPSORT

HEAPIFY $O(n)$

EXTR-MIN n times : bubbling down : $2 \log n$

total $\sim 2n \log n$

10.49 EMERGENCY EXITS

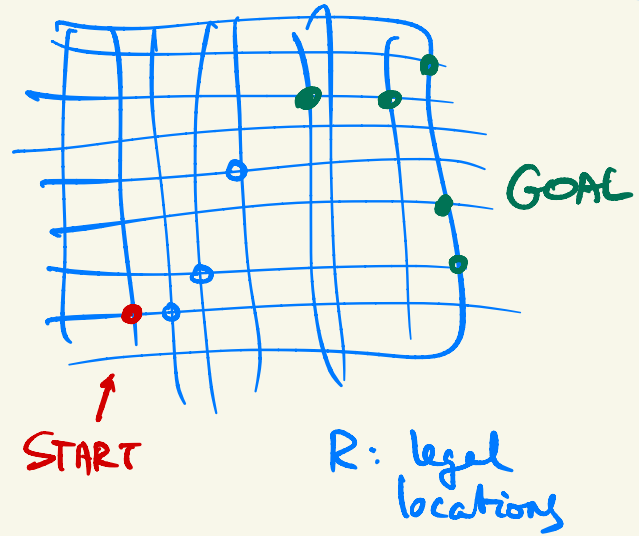
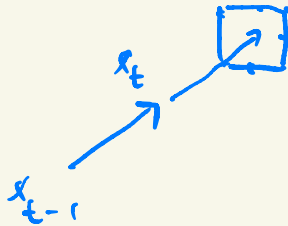


10.59 CAR RACE

(5)

$\underline{x}_t$: location at time t

$\underline{v}_t = \underline{x}_t - \underline{x}_{t-1}$: velocity



$(\underline{x}_t, \underline{v}_t)$: state

State space = $V = R \times \{\text{possible velocities}\}$
 $(2n+1)^2$

$$\deg^+ \leq 9$$

$$|V| = \underline{O(|R| \cdot n^2)}$$

(c) allows only $O(\underline{|R| \cdot n})$ time & space 6

Lemma 1 velocity: $O(\sqrt{n})$

need membership in R in $O(1)$

① if $|R| \geq n$ then take R as $n \times n$ $(0,1)$ array

② if $|R| < n$

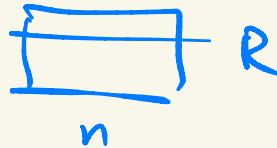
$$R = \{(x_i, y_i) \mid i \in I\}$$

$$T := \{x \mid (\exists y) (x, y) \in R\}$$

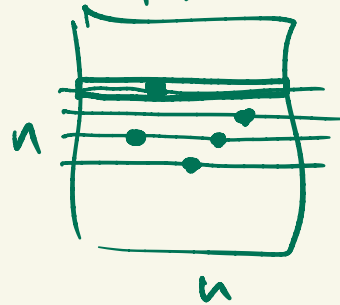
$T \subseteq [n]$ found in single pass on R

↑ array

Query (x, y)



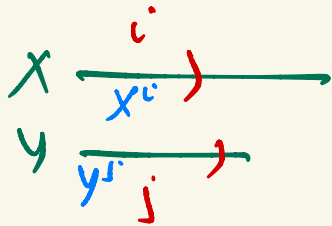
← could be $|R| \times |R|$



10.62

Edit distance: Given strings X, Y $|X|=k$
 $|Y|=l$
find $\text{dist}(X, Y)$

(7)



$$d(i, j) := \text{dist}(x^i, y^j)$$

$$d(i, j) = \begin{cases} d(i-1, j-1) & \text{if } x_i = y_j \end{cases}$$

INS
DEL
CHANGE

$$1 + \min(d(i-1, j)$$

$$d(i, j-1)$$

$$d(i-1, j-1))$$

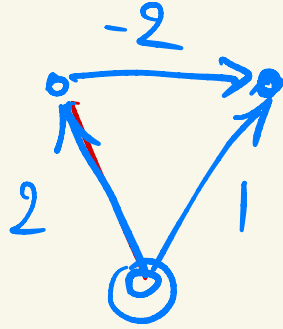
DEL y_j DEL x_i CHANGE x_i

≤

10.31

Dijkstra fails w negative weight

8



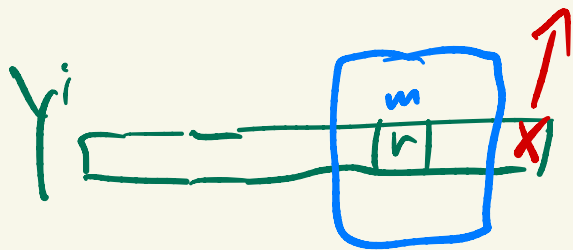
10.62 EDIT DISTANCE

x^{i-1}

9



DEL, CH



DEL

$$x_0 = x, y_0 = y$$

1) CH at $n \rightarrow "r"$

