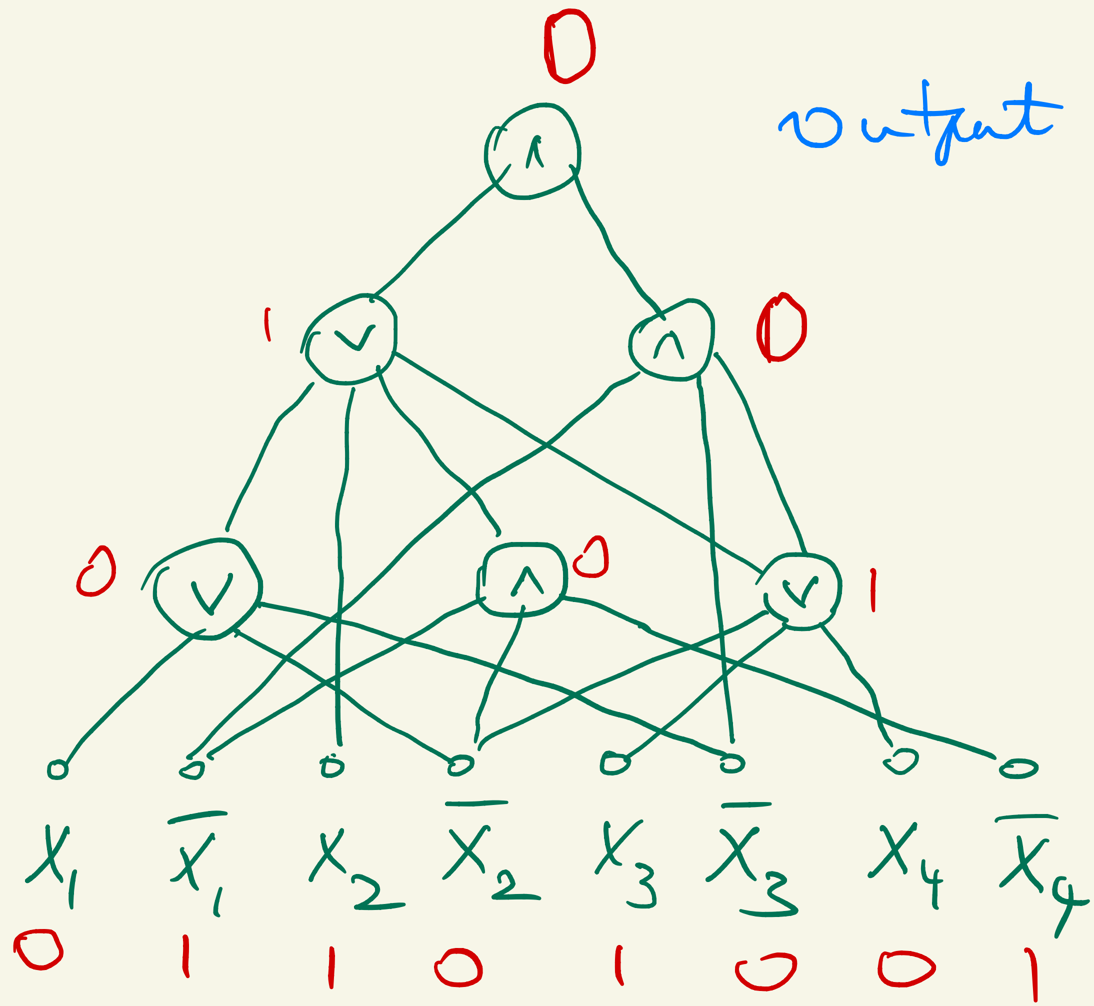
DAG
directed acyclic graph

Boolean circuit

Size = # wires

SATISFIABLE if
$(\exists \underline{x})(B(\underline{x})=1)$

output



Literals

(f_n) Boolean functions

Thm $\underline{x} \in \{0,1\}^n$ $\quad f_n(\underline{x}) \in \{0,1\}$ Boolean fctn.

If f_n computable in t steps in bit model

$\Rightarrow$ — " — by a Boolean circuit of size $O(t)$

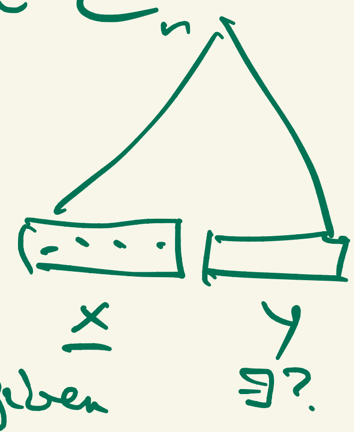
Cook-Levin Thm $\quad$ SAT $\in$ NPC

judge $B_n(x,y)$ Boolean fctn

Pf $\quad$ Q: $(\exists y)(B_n(x,y) = \text{TRUE})$ $\quad |y| \le |x|^c$

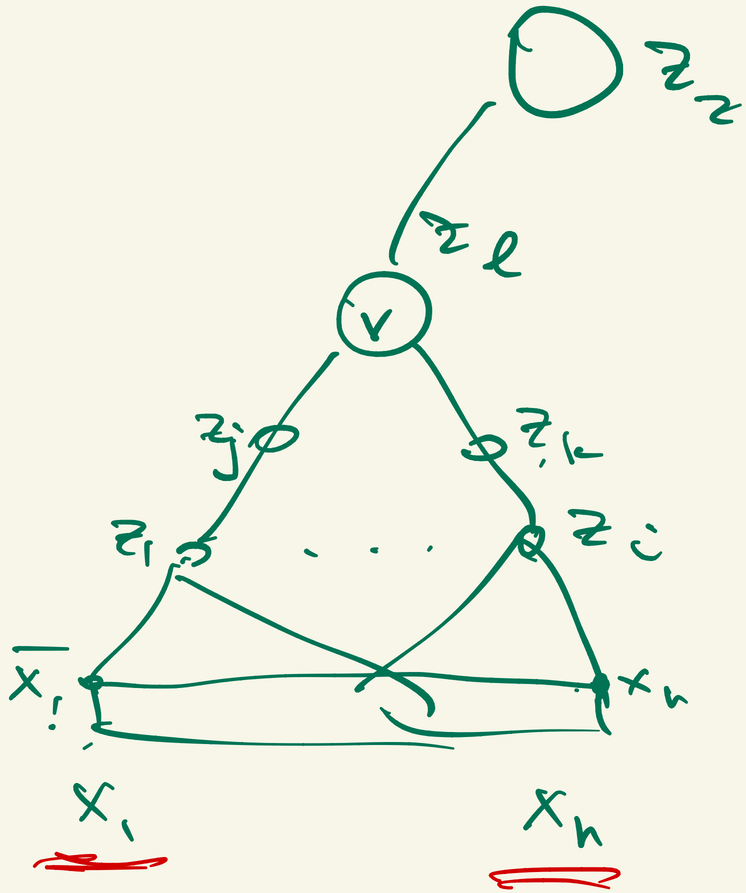
convert B_n into Boolean circuit C_n

size of C_n is $n^{O(1)}$



$\{x \mid (\exists y)(B(\underline{x},y))\} \in SAT$

↑ orig..



orig. ckt satisfiable

$\longleftrightarrow$ new ckt is

new variables:

1 per node

$$z_\ell \longleftrightarrow z_j \vee z_k$$

i.e.

$$(z_\ell \wedge (z_j \vee z_k)) \vee (\bar{z}_\ell \wedge \bar{z}_j \wedge \bar{z}_k)$$

DNF

$$(z_\ell \wedge z_j) \vee (z_\ell \wedge z_k) \vee \quad \rightarrow \quad -$$

$$a \wedge b \longleftrightarrow (a \wedge b \wedge c) \vee (a \wedge b \wedge \bar{c})$$

CNF: for negation DNF

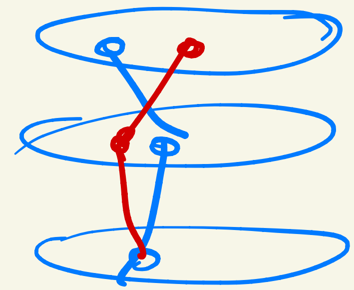
$$(\bar{z}_\ell \vee \bar{z}_j) \wedge (\bar{z}_\ell \vee \bar{z}_k) \wedge (z_\ell \vee z_j \vee z_k)$$

$$\bigwedge (\text{this for every node}) \wedge z_z$$

Thm $\quad SAT \propto_{Karp} 3SAT$

$\therefore \quad 3\text{-}SAT \in NPC$

3DM $\quad$ 3-dim matching



INPUT: $A, B, C, \; E \subseteq A \times B \times C$

Q: $\exists$ perfect matching?

$\mapsto F_1 \dots F_k \in E$ s.t. $A \cup B \cup C = \dot{\bigcup}_{i=1}^{k} F_i$

disjoint

(necessary: $|A| = |B| = |C| =: k$)

Thm $\quad 3SAT \propto_{Karp} 3DM$

SUBSET-SUM

Input: $a_1, \dots, a_n, b \in \mathbb{N}$

Question: $(\exists I \subseteq [n])(\sum a_i = b)$

3DM $\propto_{\text{Karp}}$ SUBSET-SUM $\quad\therefore$ SUBSET-SUM $\in$ NPC

<u>Pf</u> SUBSET-SUM $\in$ NP witness I

$E = (F_1 \dots F_m)$

given an <u>instance</u> of 3DM (A, B, C, E)
'input

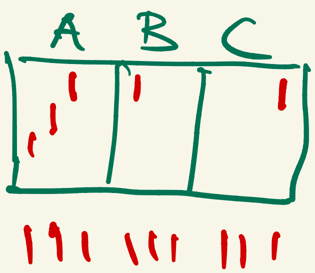
$k = |A| = |B| = |C|$

<u>create</u> an instance of <u>SubsSum</u>

$a_1 \dots a_k, b$ $\leftarrow$ write these numbers in base <u>$m+1$</u>

$k \times 3k$

$k\{$ a_1 : a_m



b

If $F_1 \dots F_k$ is a perf matching

then $a_1 + \dots + a_k = \underbrace{111111}_{m+1}$