COMBINATORICS PROBLEM SESSION

2024-03-22

(a_n) sequence of reals

$$\lim_{n\to\infty} a_n = L \qquad L \in \mathbb{R}$$

$$(\forall \varepsilon > 0)(\exists n_0)(\forall n)\,(n \geq n_0 \Rightarrow |a_n - L| < \varepsilon)$$

"for all sufficiently large n"

$\underbrace{\forall \quad \exists}_{\text{quantifiers}}$

A E

$\limsup a_n = L$

X $\quad \sup a_n = L$

$1, 0, 0, \ldots$

$\frac{1}{n}$

$\sup = 1$

$\limsup = 0$

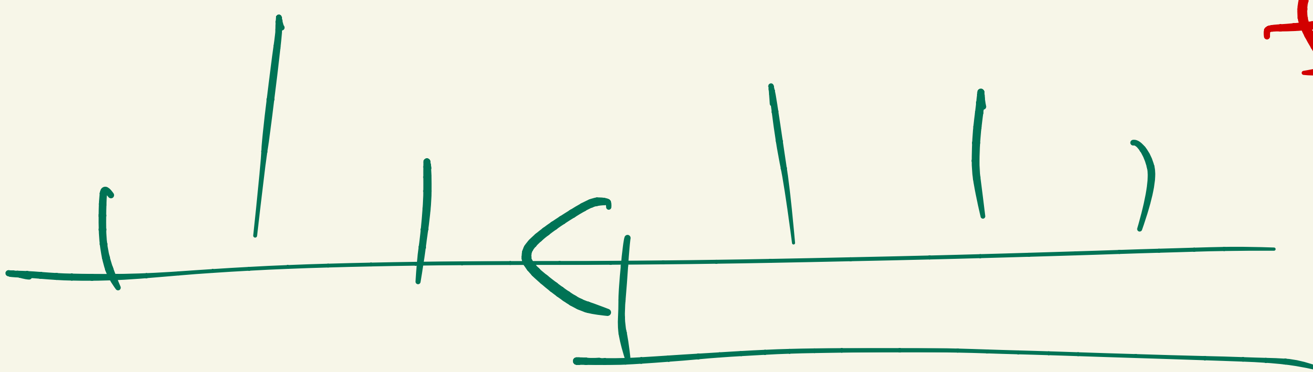
tail predicate

predicate on Ω

$f : \Omega \to \{0,1\}$

F/T

N/Y

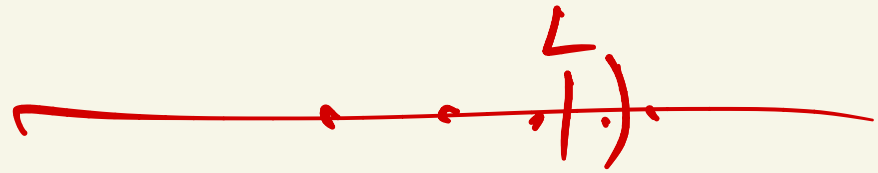


$A_n = \sup(a_i \mid i \geq n)$

$A_n \geq A_{n+1} \geq \ldots$

$\lim A_n = L$

① $\limsup a_n \le L$ $\quad (\forall \varepsilon > 0)(\exists n_0)(\forall n \ge n_0)(a_n \le L + \varepsilon)$



$\boxed{3}$

② $\ge L$ $\quad (\forall \varepsilon > 0)$ inf. often $a_n \ge L - \varepsilon$

$(\forall n_0)(\exists n \ge n_0)(\quad)$

~~1~~ $\limsup a_n \le L \iff (\forall n)(\exists m)\underbrace{(m \ge n \wedge a_m \le L)}_{\text{infinitely often } a_n \le L}$

such that

AND

③ counterexample to

3a $\not\Rightarrow$ $\frac{1}{n}$

$\limsup \frac{1}{n} = 0$

3b $\Leftarrow$ $0 \; 1 \; 0 \; 1 \; 0 \; 1 \ldots$

$L = 0$ $\quad \limsup = 1$

$$\underbrace{(\forall x)}_{x \in \mathbb{N}_0}(\exists y, z)\underbrace{(x = y^2 + z^2)}$$

$\mathbb{N}_0$ | 4

$\mathbb{R} \geq 0$

$$\neg \; \forall x \; \exists y \; f(x, y)$$

predicate

$$\exists x \; \forall y \; \neg f(x, y)$$

$\mathbb{N}_0 = \{0, 1, 2, \dots\}$

$(\forall x)(\exists u, v, w)(x = u^2 + v^2 + w^2)$

$(\forall x \not\equiv 7 \bmod 8)$ — " —

$\mathbb{Z}$

7

15

congruence

$a \equiv b \bmod m$

if $m \mid a - b$

↑ divides

$a \mid b$ if $(\exists x)(ax = b)$

$2 = 1^2 + 1^2 + 0^2$

$3 = 1^2 + 1^2 + 1^2$

$8 = 2^2 + 2^2 + 0^2$

$11 = 3^2 + 1^2 + 1^2$

$\mathbb{N}_0$

$$(\forall x)(\exists p, q, r, s)(x = p^2 + q^2 + r^2 + s^2)$$

TRUE LAGRANGE: 4-square thm

a, b days of month

when do they fall on same day of the week?

$a \equiv b \mod 7$

2^{nd} 23^{rd}

a, b are congruent modulo 7

$2 \equiv 23 \mod 7$

b/c $7 \mid 2 - 23 = -21$

Lin Alg "Algebra"

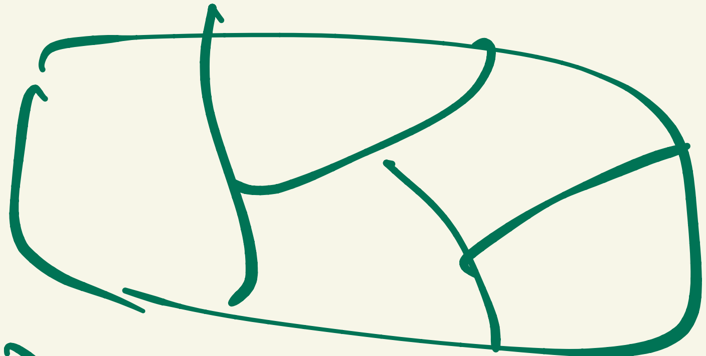
congruence mod m is an equivalence relation among integers

reflexity $a \equiv a \pmod m$

symmetry $a \equiv b \Rightarrow b \equiv a$ "

transitive $a \equiv b \wedge b \equiv c \Rightarrow a \equiv c$

Partition of Ω



$$\Pi = (A_1, \ldots, A_k)$$

$$A_i \subseteq \Omega$$

$$A_i \neq \emptyset$$

$$i \neq j \Rightarrow A_i \cap A_j = \emptyset$$

$$\bigcup A_i = \Omega$$