HONORS ALGORITHMS
PROBLEM SESSION

2025-01-10

p1

ASY !

REAS 22

DEF $a_n \sim b_n$ if $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$

DEF (c_n) is eventually nonzero
if

S: set of sequences

$\forall$ A
$\exists$ E

(A) $(\forall M)(\exists n)(n > M \wedge c_n \neq 0)$

(B) $(\exists M)(\forall n)(n > M \Rightarrow c_n \neq 0)$

threshold

means in plain English

(A) [infinitely many are non-zero]

(B) [only finitely many are zero]

prime counting function

$\pi(x) = \#\text{primes} \le x$

PNT $\quad \pi(x) \sim \dfrac{x}{\ln x}$

p_n : n^{th} prime number

challenge $\quad \boxed{p_n \sim n \ln n}$

$\dfrac{\ln n}{n} \to 0$

$\dfrac{n}{e} < n$

$\dfrac{n}{\log\log n} < n$

$\ln n$

$\log 2n$

$\log^2 n$

n^2

$\pi^{-1}(n)$

$\pi(n) \cdot \ln n \sim n$

$N \cdot \dfrac{n}{\ln n}$

$n \cdot \dfrac{n}{\ln n} = \dfrac{n^2}{\ln n}$

e^n

P3

$$P(x) = \prod_{\substack{p \le x \\ p \text{ prime}}} p \approx e^x$$

Chall.

$$\ln P(x) \sim x \longleftrightarrow \text{PNT}$$

x power 2 $\qquad$ x^2

$$x^{\pi(x)} \approx x^{\frac{x}{\ln x}} = e^x$$

$$P(x) < e^{x(1+\varepsilon)}$$

$$P(x) > e^{x(1-\varepsilon)}$$

$$\ln P(x) < x(1+\varepsilon)$$

$$> x(1-\varepsilon)$$

$\forall \varepsilon > 0$

$\exists M$

$\forall x \ge M$

$$1-\varepsilon < \frac{\ln P(x)}{x} < 1+\varepsilon$$

p4

$$f(n) = \underline{5n^7} - 1000n^6 + 7n^3 - 10^6$$

$$f(n) \sim 5n^7$$

$$\frac{f(n)}{5n^7} = 1 - \frac{c_1}{n} + \frac{c_2}{n^4} - \frac{c_3}{n^7} \to 1$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$0 \quad 0 \quad 0$$

$$\sqrt{n^2+1} - n \to \cancel{0} \qquad \varepsilon(n)$$

$$2^n + n \quad \cancel{\to} \quad 2^n$$

$$\sim$$