HONORS ALGORITHMS | 2025-01-29

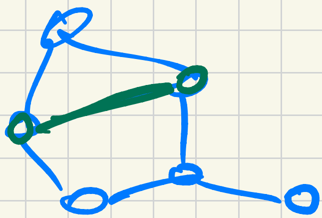
p1

DM mini Chap 6.1 "GRAPH TERMINOLOGY"

$G = (V, E)$ $\quad E \subseteq \binom{V}{2}$

subgraph $H = (W, F)$

$H \subseteq G$ if $W \subseteq V$, $F \subseteq E$



x, y are adjacent / neighbors

deg(x) = # neighbors of x

$n = |V|$

$m = |E|$

HANDSHAKE THM:

$$\sum_{x \in V} \deg(x) = 2m$$

COR: # vertices of odd degree is even

<u>Complement</u> $\overline{G} = (V, \overline{E})$

$(\forall x,y \in V)(x \sim_{\overline{G}} y \iff x \not\sim_G y \wedge x \neq y)$

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an isomorphism $f : G \to H$
is a bijection $f : V \to W$
s.t. $(\forall x,y \in V)(x \sim_G y \iff f(x) \sim_H f(y))$

$G \cong H$ <u>isomorphic</u> if $\exists G \to H$ isomorphism

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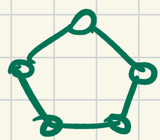
$x \in V$, $e \in E$, $x \in e$
then we say x, e are <u>incident</u>

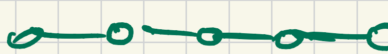
K_n complete graph $(V, \binom{V}{2})$

C_n cycle of length n



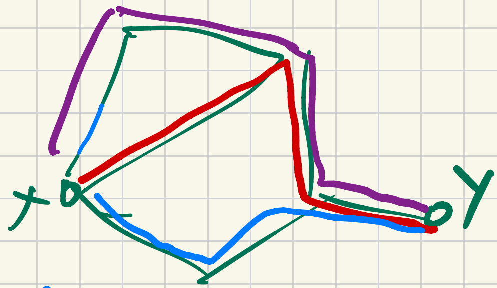
C_5

P_n path of length $n-1$



P_5

$x, y \in V$



y is accessible from x

is $\exists\ x \cdots y$ path

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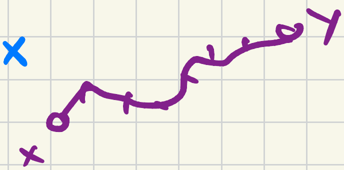
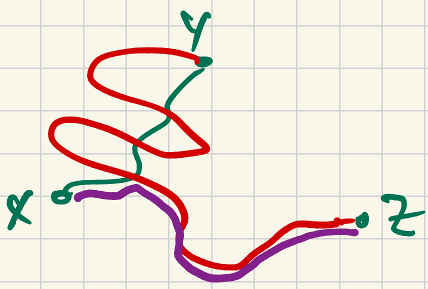
accessibility is an equivalence relation on V

reflexive: x acc. x $\quad P_1$: length 0

symm: x acc $y \Rightarrow y$ acc x



transitive:

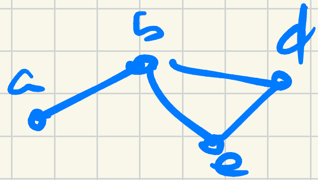


DEF Walk of length k in G

$$x_0 - x_1 - \cdots - x_k$$

where $(\forall i \in [k])(x_{i-1} \sim x_i)$

Lemma If $\exists\, x \cdots y$ walk $\Rightarrow \exists\, x \cdots y$ path



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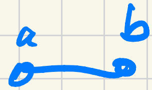


ababab

EX

Proof 1: Claim:

$\forall$ shortest walk is a path (disregard orientation)

Proof 2: efficient alg. – linear time

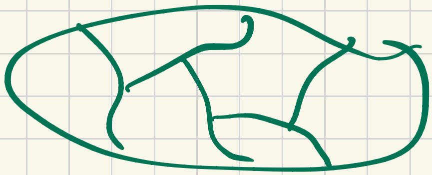
PARTITION of a set V:

$$V = W_1 \sqcup W_2 \sqcup \cdots \sqcup W_k \qquad W_i \neq \emptyset$$

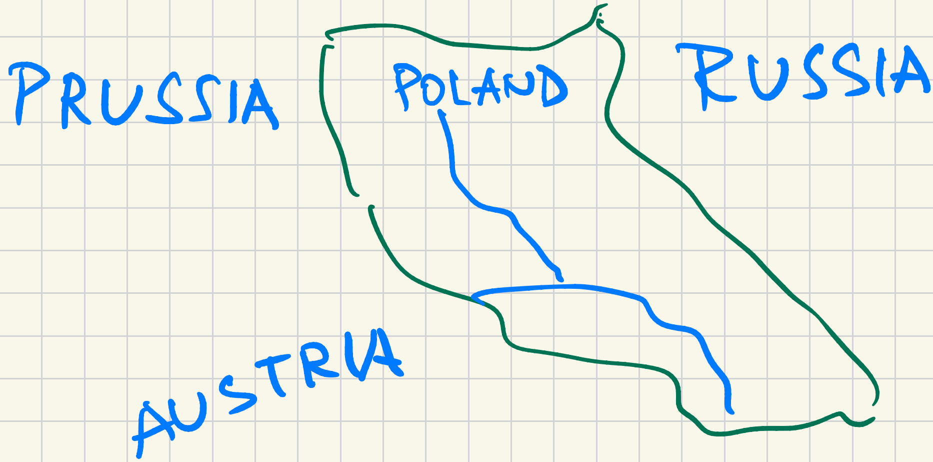
$\sqcup$ disjoint union:

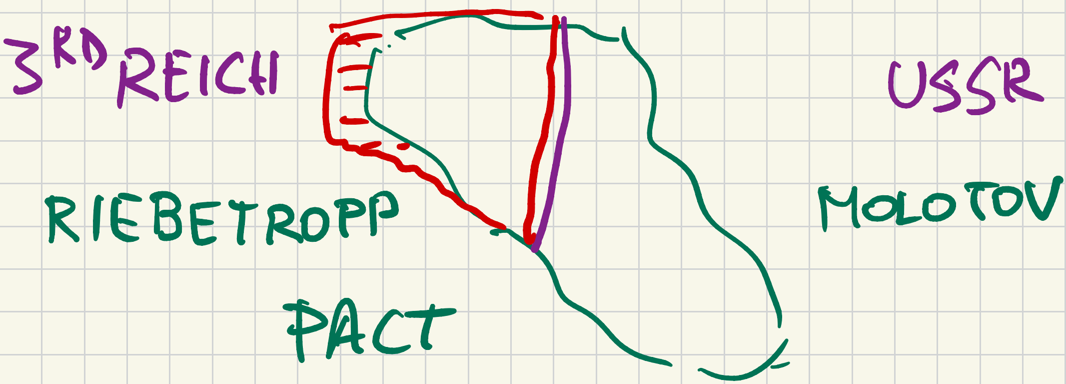
$V = W_1 \cup \cdots \cup W_k$

and $(\forall i \neq j)(W_i \cap W_j = \emptyset)$

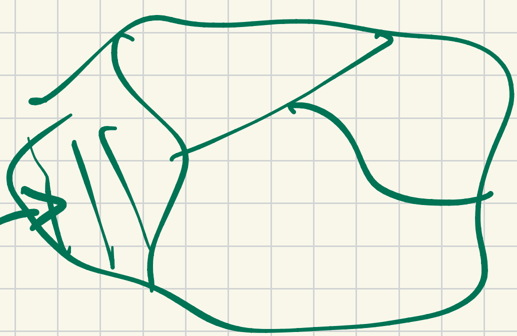


V

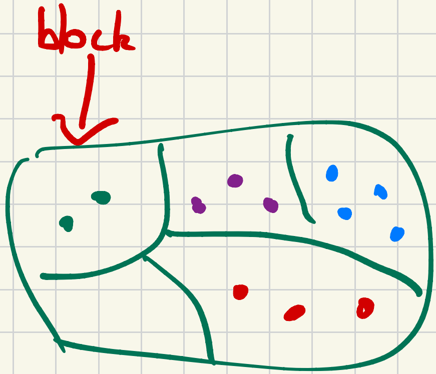




Partition
$\{W_1, \ldots, W_k\}$



a PART



$$\sqcup \quad \sqcup \quad \sqcup$$

- FUNDAMENTAL THM OF EQ. REL'S

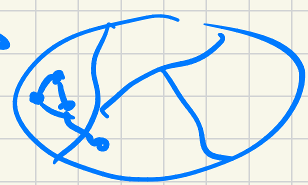
Every eq. relation arises from a unique partition | of intelligence

Equivalence classes

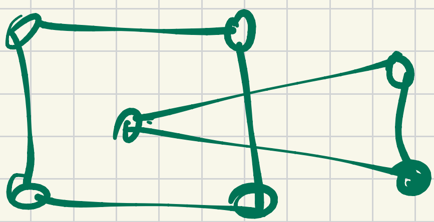
$$\frac{4}{6} \sim \frac{10}{15} \quad \ldots \quad \frac{a}{b} \sim \frac{c}{d}$$

if $ad = bc$

blocks: rational numbers



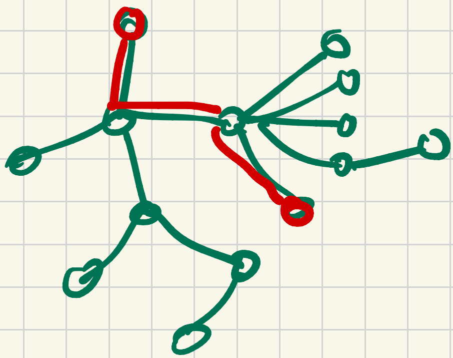
equivalence classes of the accessibility relation on a graph: "connected components"



DEF G is <u>connected</u> if just 1 connected component

i.e. $(\forall x, y \in V)(\exists\ x \dots y$ path$)$

tree : cycle-free connected graph



$\boxed{\text{DO}}$ G is a tree $\iff$

$(\forall x, y \in V)(\exists!\ x \cdots y \text{ path})$

"unique": exactly one

modern IT usage

"unique" : distinct

$\boxed{\text{DO}}$ If G is a tree then $m = n-1$

rooted tree