HONORS ALGORITHMS

2025-02-05

p1

DIJKSTRA's algorithm

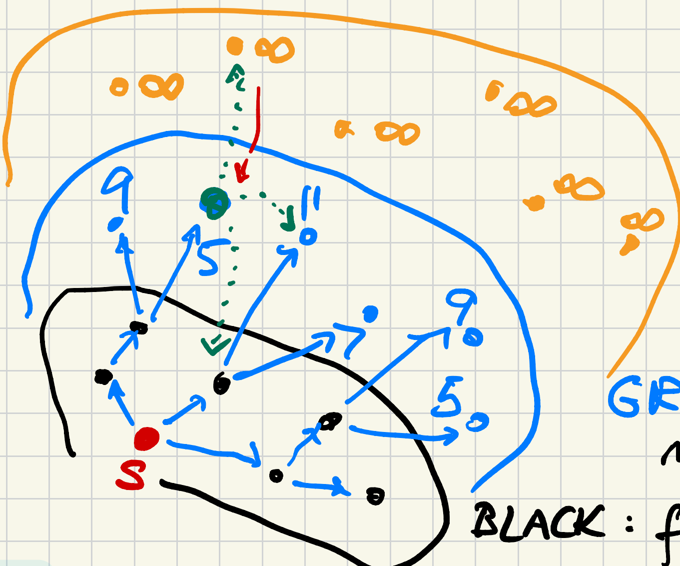
INPUT: (V, E, w, s)

(V, E) digraph, $w : V \to \mathbb{R}$

$(\forall x \in V)(w(x) \geq 0)$

$s \in V$ "source"

SINGLE SOURCE MIN-WEIGHT PATH problem ("SHORTEST")



WHITE: undiscovered

GRAY: discovered not finished

BLACK: finished

for $v \in V$ **do** $\qquad$ q : weighted distance

init

$q(v) = \infty$, $p(v) = NIL$, status = white, $Q = \emptyset$

$q(s) = 0$, $p(s) = s$, status(s) = grey

Q queue — stores gray vertices

add s to Q

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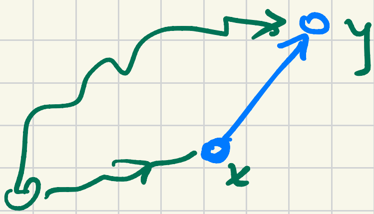
while $Q \neq \emptyset$ **do**

$x \leftarrow$ EXTRACT-MIN(Q)

for $y \in adj(x)$ $\qquad$ y : out-neighbor of x

RELAX(x, y)

status(x) := BLACK



RELAX(x, y)

if status(y) = WHITE

then status(y) := GRAY

INSERT(Q, y)

$p(y) := x$; $q(y) := q(x) + w(x, y)$

elseif status(y) = GRAY

then **if** $q(y) > q(x) + w(x, y)$

then $q(y) :=$ -"- , $p(y) := x$

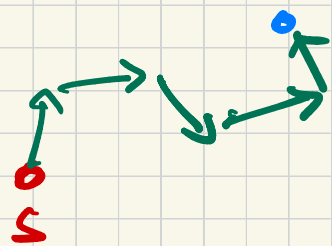
PRIORITY QUEUE

EXTRACT-MIN(Q)
INSERT(y)
DECREASE-KEY(y, newkey)

$n = |V|$
$m = |E|$

Cost: EXTRACT-MIN $n_0 \leq n$

\# accessible vertices from source

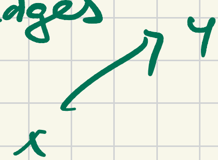


INSERT n_0 times $\leq n$

DECREASE-KEY $\leq \underline{m_0}$ times $\leq m$

\# accessible edges

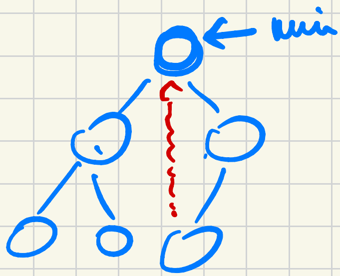
(x, y) accessible if
x is – " –



p4

first implementation of PRIORITY QUEUE

HEAP



cost of each of the three data str operation $O(\log n)$

total cost $O((n+m)\log n)$

THM Fredman-Tarjan ≈ 1980

Dijkstra can be executed in

$$O(n \log n + m)$$

linear if $m = \Omega(n \log n)$

FREDMAN-TARJAN

k	INSERT	$O(1)$	$\times\ n$
ℓ	EXTACT-MIN	$O(\log n)^*$	$\times\ n$
m	DECREASE-KEY	$O(1)^*$	$\times\ m$

total $O(n \log n + m)$

* AMORTIZED COMPLEXITY

start $Q = \{s\}$

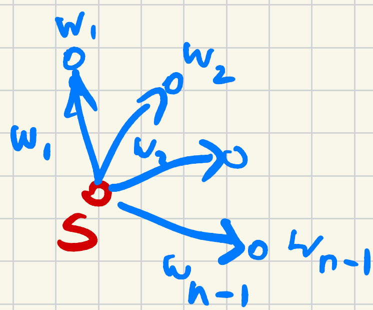
s.t. Q is never empty

total cost of ~~any~~ sequence of queries is $O(k + \ell \cdot \log n + m)$

Fibonacci heap

Claim Cost of impl of Dijkstra $\Omega(n \log n)$ in the sense that $(\forall m \geq n-1)(\exists \text{ digraph})$(on this digraph, Dijkstra simulates sorting by comparisons of $n-1$ data)

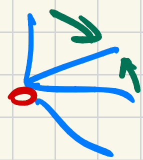
First assume $m = n-1$



finalized in sorted order

$\therefore$ cost $\geq n \log n$

$m \geq n-1$



weight: $1 + \max w_i$

cost: $\geq (n-1)\log(n-1) + m$