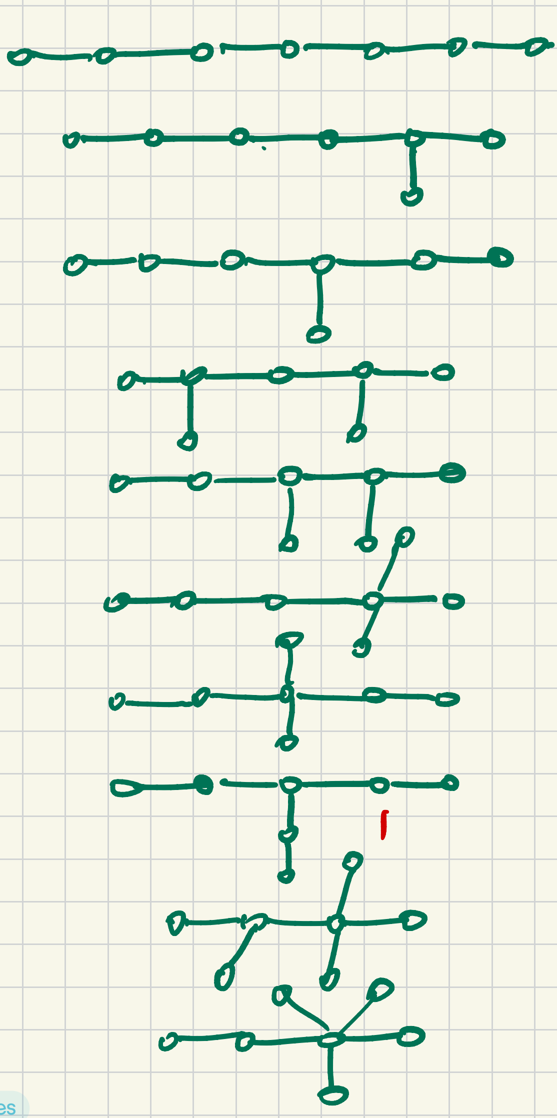
H. ALGORITHMS PROBLEM SESSION | 2025-02-07

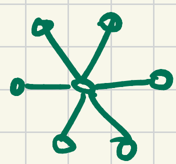
p 1

9.28 7-vertex trees

P_7



11



p2

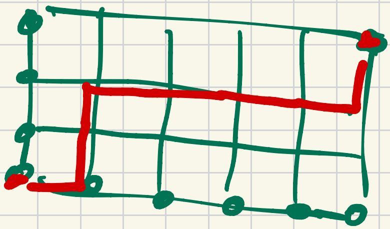
9.22

spanning subgraphs of K_n with m edges

$$\binom{\binom{n}{2}}{m}$$

9.37

k



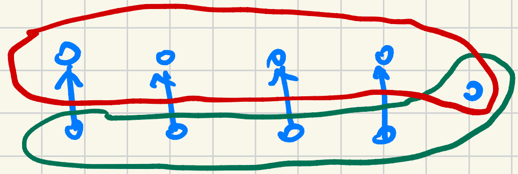
ℓ

$(k-1)+(\ell-1)$
$= k+\ell-2$

$$\binom{k+\ell-2}{k-1}$$

7.67 MIN & MAX

$\leq \frac{3n}{2}$ comparisons



$\lfloor \frac{n}{2} \rfloor$ comparisons

$\lceil \frac{n}{2} \rceil$ candidates remain

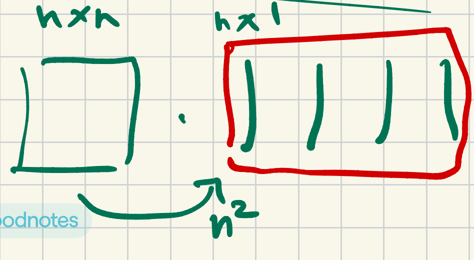
find min among them

$\lfloor \frac{n}{2} \rfloor + 2\left(\lceil \frac{n}{2} \rceil - 1\right)$ comparisons

if n even $\frac{3n}{2} - 2$

if n odd

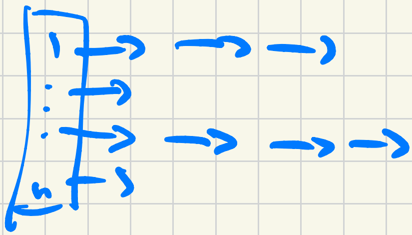
$$\frac{n-1}{2} + 2 \cdot \frac{n+1}{2} - 2 = \frac{3n}{2} - \frac{3}{2}$$



n^3

$n^{\log_2 7}$

adj array: array of adj lists



edge list $[(x_1, y_1)(x_2\, y_2) \cdots (x_m\, y_m)]$

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edge list → adj array

<u>for</u> $i=1$ <u>to</u> m

    <u>append</u> y_i <u>to</u> adj$[x_i]$

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adj array → edge list

<u>for</u> $i=1$ <u>to</u> n

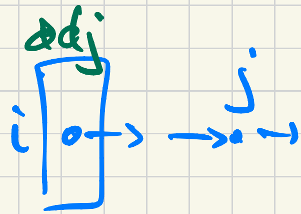
    <u>for</u> $j \in$ adj(i)

        <u>append</u> (i,j) list list

$G = (V, E) \quad \bar{G} = (V, \bar{E})$ (p5

2nd sol'n — reverse easiest edge-list

for $i = 1$ **to** m
  append (y_i, x_i) **to** $\bar{E}$



1st sol'n

for $i = 1$ **to** n
  for $j \in adj[i]$
    append i **to** $\bar{adj}[j]$

$G \longrightarrow G^{\text{monotone}}$

$adj[i] = [j_1 \cdots j_k]$

$j_1 < j_2 < \cdots < j_k$

✗ Sorting line by line

by comparisons: $\sum d_i \log d_i$ $\qquad \sum d_i = 2m$

$2m \log d_{min} < \quad < 2m \cdot \log d_{max}$

not $O(1)$

counting sort

$$\sum (d_i + n) = 2m + n^2 = O(n^2)$$

1st sol

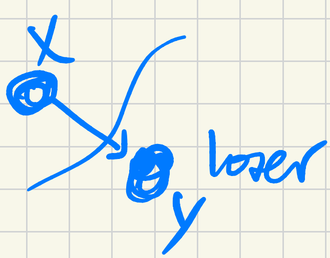
reverse (reverse (G))

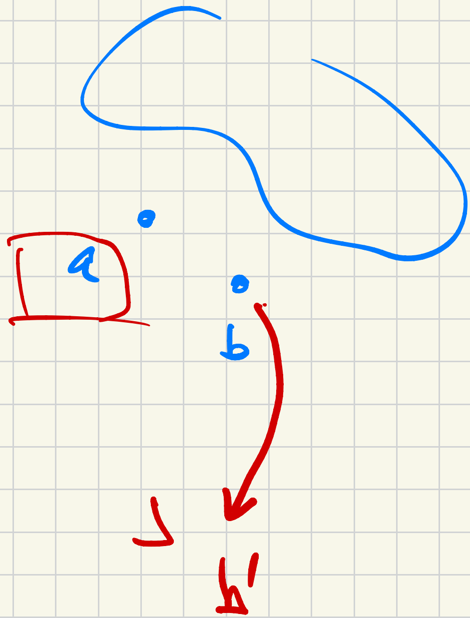
2nd

adj list → edge list → radix sort → adj list

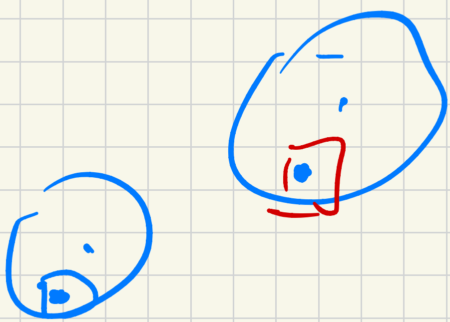
7.64 FIND·MIN

candidate: has not yet been beaten



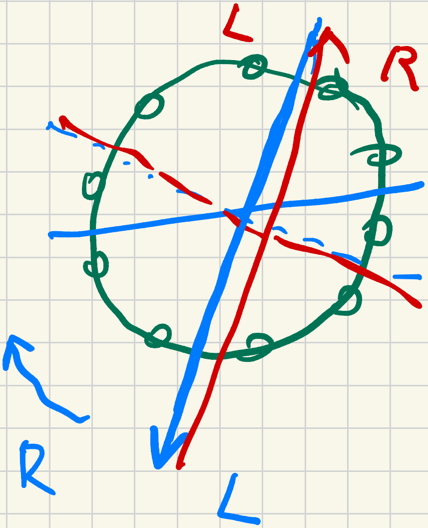


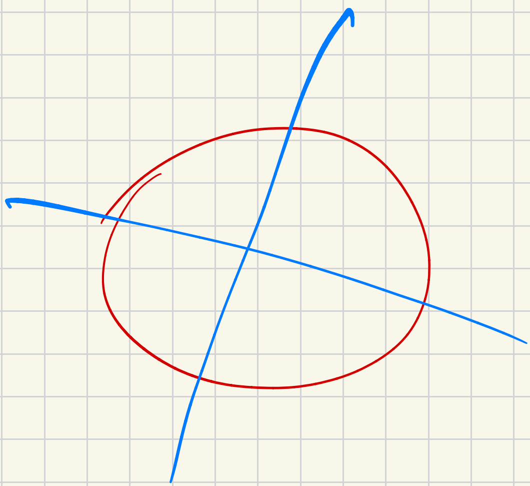
thm connected graph has $\geq n-1$ edges

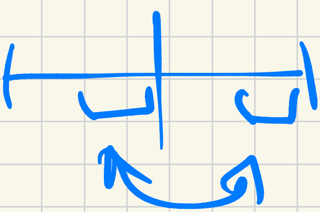


7.61 evenly splitting set of fake coins

n coins, fake < genuine







7.39 $T(n) \le T\left(\lfloor \frac{n}{2} \rfloor\right) + T\left(\lceil \frac{n}{2} \rceil\right) + \underbrace{n-1}_{\text{merge}}$

Claim $T(n) \le n \log_2 n$

$n = 2^k$ $T(n) \le 2T\left(\frac{n}{2}\right) + n - 1$

Pf find $g(n) \ge 2g\left(\frac{n}{2}\right) + n - 1$

$g(1) \ge T(1) = 0$

$g(n) := n \log_2 n$

$$g(n) = n \log_2 n \overset{?}{\ge} 2\,\frac{n}{2}\log\frac{n}{2} + n - 1$$

$n \ge 2$

$$n(\log n - 1) + n - 1$$

$$n \log n - 1$$

$S(n)$ incl bookkeeping

$$S(n) \le S\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + S\left(\left\lceil \frac{n}{2} \right\rceil\right) + O(n)$$

Cn

need $g(n) \ge g\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + g\left(\left\lceil \frac{n}{2} \right\rceil\right) + Cn$

$g(1) > 0$

even

$g(n) \ge 2g\left(\frac{n}{2}\right) + Cn$

$g(n) = \underline{An\log n + Bn}$

$$An\log n + Bn \overset{?}{\ge} A\left\lfloor \frac{n}{2} \right\rfloor \log\left\lfloor \frac{n}{2} \right\rfloor + A\left\lceil \frac{n}{2} \right\rceil \cdot \log\left\lceil \frac{n}{2} \right\rceil + Cn$$

$$\underbrace{+B\left\lfloor \frac{n}{2} \right\rfloor + B\left\lceil \frac{n}{2} \right\rceil}$$

for n even

$$\underline{An\log n} + Bn \overset{?}{\ge} 2A\frac{n}{2}\log\frac{n}{2} + 2B\frac{n}{2} + Cn$$

$$\underline{An(\log n - 1)}$$

$$Bn \overset{?}{\ge} -2An + Bn + Cn$$

$$\boxed{2A \overset{?}{\ge} C}$$

Ok for $n = 2^k$

general case

$2^{k-1} < n \le 2^k$

$\boxed{S(n) \le} \boxed{\underline{S(2^k)} \le \underline{A\, 2^k \cdot k}} \le 4An\log_2 n + 1$

$\overset{?}{\le} 4 A n \log_2 n$

b/c $n > 2^{k-1}$

$\underline{2n > 2^k}$

need: $4n \log_2 n \overset{?}{\ge} 2^k \cdot k$

$\log_2 n \ge k-1$

$4n \log_2 n =$

$2 \cdot 2n \cdot \log_2 n \ge$

$2 \cdot 2^k \cdot (k-1)$

need $2(k-1) \ge k$

$k \ge 2$ ✓

Alternative solution

$$g(n) : \begin{cases} A n \log n & n \text{ even} \\ g(n+1) & n \text{ odd} \end{cases}$$