ALGORITHMS PROBLEM SESSION

2025-02-14

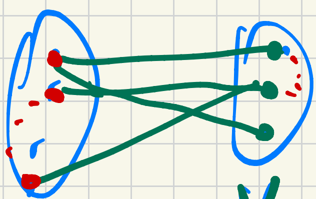
p1

bipartite graph $G = (V, E)$

s.t. $V = V_1 \sqcup V_2$ and
$\exists V_1, V_2$

disjoint union

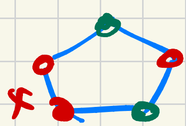
every edge connects vertices from different parts



V_1 V_2

Obvious: if G is bipartite then every subgraph " "

Odd cycle NOT bipartite



$\therefore$ Bipartite graph has no odd cycles

Thm: G is bipartite $\iff$ G has no odd cycles

$\boxed{\text{TODO}}$

$\Rightarrow$ ✓

$\Leftarrow$

hint: spanning tree

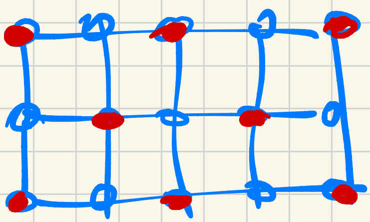
Good characterization:

$\exists$ something $\iff$ $\nexists$ sg else

poly-time verifiable

$\exists$ 2-coloring $\iff$ $\nexists$ odd cycle

EXAMPLE



Grid (3,5)

Claim:
$\forall$ grid is bipartite

"checkerboard coloring"

p 3

9.43

$k \times l$ grid, k, l odd $\Big\} \Rightarrow$ not Hamiltonian

– no H-cycle (cycle of length n)

$n = \underbrace{kl}_{\text{odd}}$

odd cycle : cannot be b/c grid is bipartite

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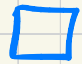
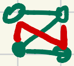
kl even $\Rightarrow \exists$ H-cycle

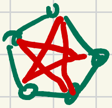
assuming $k, l \geq 2$



9.18 self-complementary $G \cong \bar{G}$

$\Longrightarrow n \equiv 0$ or $1 \mod 4$

P_4 

C_5 

$$m_G = m_{\bar{G}}$$

$$\underbrace{m_G + m_{\bar{G}}}_{2m_G} = \binom{n}{2} = \frac{n(n-1)}{2}$$

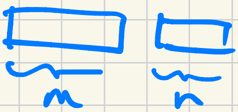
$$m_G = \frac{n(n-1)}{4} \begin{cases} 4 \mid n \\ 4 \mid n-1 \end{cases}$$

b/c other is odd

\# comparisons

MERGE $\underline{\text{optimal}}$ if $m=n$

$m+n-1$ comparisons



far from opt. if $m = o\left(\frac{n}{\log n}\right)$

$m=n$ req. $2n-1$ comparisons

NTS $2n-2$ don't suffice

meaning: if algor makes only $2n-2$ comparisons then $\exists$ input that result in wrong output

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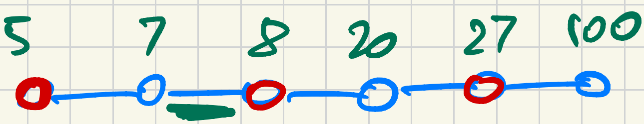
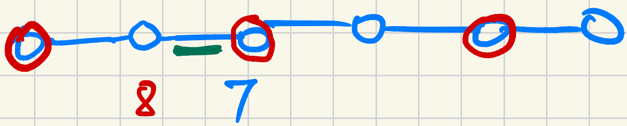
INPUT : RED LIST, BLUE LIST



← not compared

INPUT: 5, 8, 27   7, 20, 100

NEW INPUT: 5, 7, 27   8, 20, 100

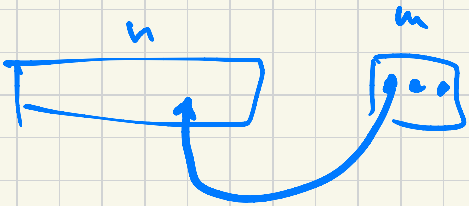


gives same answer to each comparison

→ $\underline{\text{incorrect output}}$

7.28

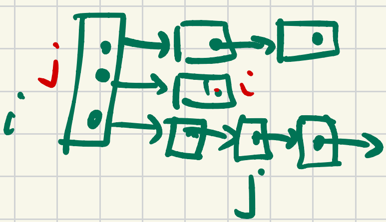
MERGE by INSERTS



$$m \cdot \log n \overset{\text{is}}{=} o(m+n)$$

$$\text{iff} \quad m = o\left(\frac{n}{\log n}\right)$$

11.38 undirectedness test



G^{mon}

G^{-mon}

14.19 L'Hospital reverse ×

$f, g \longrightarrow \infty$ as $x \longrightarrow \infty$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L \qquad \lim_{x\to\infty} \frac{f'(x)}{g'(x)} = M$$

L'H: if $\exists M$ then $\exists L$ and $L=M$

reverse:

if $\exists L$ then $\exists M$ ×

Counterexample:

differentiable fctn f, g

s.t. $f, g \longrightarrow \infty$

$\exists \lim \frac{f}{g}$ $\qquad \nexists \lim \frac{f'}{g'}$

$f(x) = x + \cos x$

$g(x) = x$

$\lim \frac{f}{g} = 1$

$\frac{f'}{g'} = \frac{-\sin x}{\quad}$ $\cancel{\to}$

14.51 # bits

$b(F_k) \sim ck$ $\quad c = ?$ $\log_2 \phi$

$F_k \sim \frac{\phi^k}{\sqrt{5}}$ $\quad \phi = \frac{1+\sqrt{5}}{2}$

golden ratio

$b(F_k) = \lceil \log_2(1+F_k) \rceil \underset{*}{\sim} \log_2 F_k =$ $\quad 1+F_k \sim F_k$

$= k \log_2 \phi - \log_2 \sqrt{5} \sim k \cdot \log_2 \phi$

14.54

F_k $\quad k = \underbrace{1011101}_{\ell \text{ bits}}$

$b(F_k) = \Theta\left(C^{b(k)}\right)$

$C = ?$ $\quad 2^{\ell-1} \leq k < 2^{\ell}$

$k = \Theta(2^{\ell})$

$\sim ck$

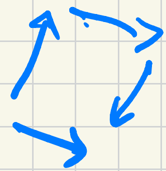
$c = \log_2 \phi$

$\ell = b(k)$

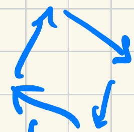
$C = 2$

13.47 Dijkstra fails on DAG vs 1 negative edge

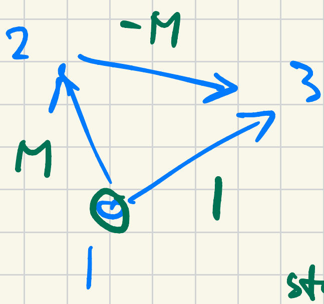
directed acyclic graph



DAG



not a DAG



	1	2	3
status	G	W	W
p	G	NIL	NIL
q	0	∞	∞
status	B	G	G
p	1	1	1
q	0	M	1
status	B	G	B
p			
q	0	M	1

B	B	B
0	M	1