H. ALGORITHMS | 2025-02-21
PROBLEM SESSION

$\forall c$

$$c^n < n! <$$

$$\exp(\Omega(n^{c_1})) < f(n) < \exp(O(n^{c_2}))$$

$$\Omega\left(c_1^{n^{c_1}}\right) \leq f(n) \leq O\left(c_2^{n^{c_2}}\right)$$

c_1: any $c_1 > 1$ valid | $\forall c_2 > 1$ | $\forall c_2 > 1$

$(c_1, c_1) = (1, c_1)$ $c_1 = 1$

$$\left(\frac{n}{e}\right)^n < n! < n^n = e^{n \ln n}$$

$c_1 > 1$ does not work b/c $c^{n^{c_1}} > e^{n \ln n}$ b/c $n \ln n = o(n^{c_1})$

Find simply exp fctn

$$\neq \Theta(C^n)$$

for any C

$$f(n) = \begin{cases} 3^n & n \text{ even} \\ 2^n & n \text{ odd} \end{cases}$$

P^3

loop invariants

R_2^*: $(\forall u \in V)\,(u \text{ is WHITE} \Leftrightarrow q(u) = \infty)$

$\Rightarrow$

u WHITE $\Rightarrow$ $q(u) = \infty$

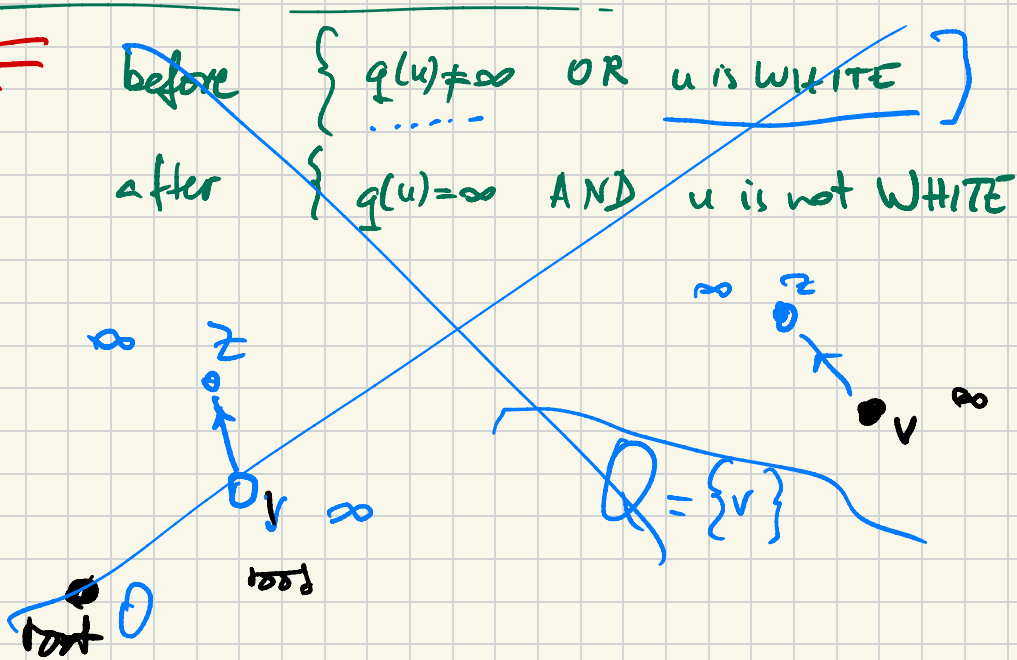
↑ needs to be true

↑ needs to be false } after

$\Leftarrow$

before { $q(u) \neq \infty$ ..... OR u is WHITE }

after { $q(u) = \infty$ AND u is not WHITE

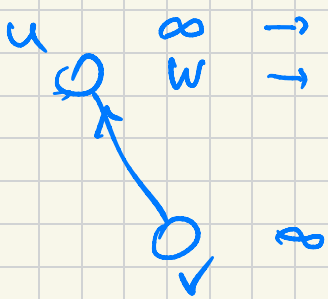


R_2^* : $(\forall u \in V)\,(u \text{ is WHITE} \iff q(u) = \infty)$

$\Rightarrow$

in: $\forall u$ { u is WHITE & $q(u) = \infty$; OR u not " & $q(u) \neq \infty$

out $\exists u$ { u WHITE & $q(u) \neq \infty$; OR u not " & $q(u) = \infty$



$Q = \{v\}$

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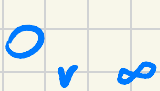
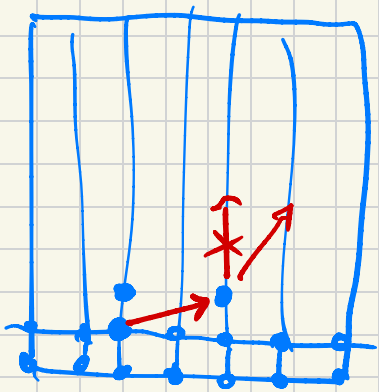
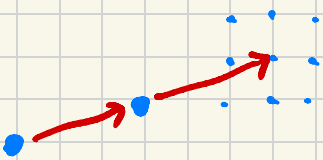




$|\Phi| = O(|R| n^2)$

$|\Phi| = O(|R| \cdot n)$

assuming $|R| = \Omega(n)$



Lemma $v = (v_1, v_2)$

$|v_i| < \sqrt{2n}$

Φ phase space

$(\underline{x}, \underline{v}) \Leftrightarrow q$

BFS takes $O(\Phi)$

$k \geq k-1 \geq k-2 \ldots$

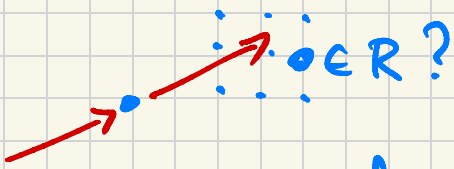
$n \geq \text{sum} \geq \binom{k+1}{2}$

$k^2 < k(k+1) \leq 2n$

$k < \sqrt{2n}$

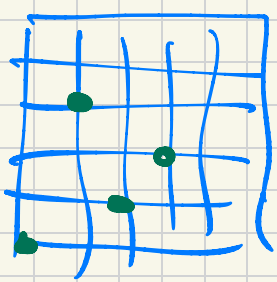
v vectors $= O(n)$

To construct phase space:



need $O(1)$ lookup

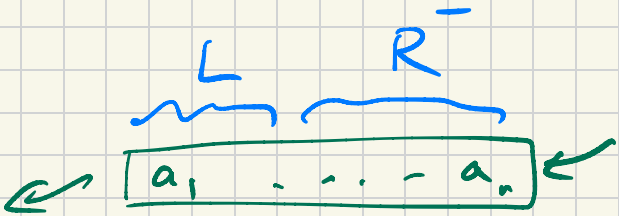
$n^2 = O(|R| \cdot n)$ time and space



need n^2 time & space

we need $C|R|n \geq n^2$

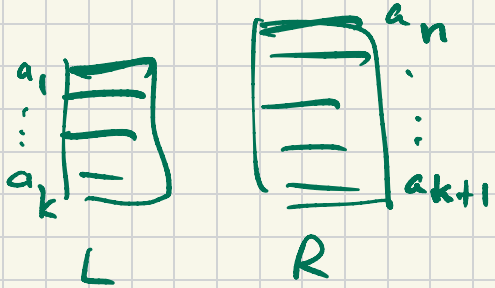
i.e. $|R| = \Omega(n)$ ✓



queue

enqueue 4

dequeue 0



pop

push

enqueue: push L

dequeue: pop R ← except if empty then pour L to R

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lifecycle of an item:

1 push L ← enqueue

1 pop L } pour

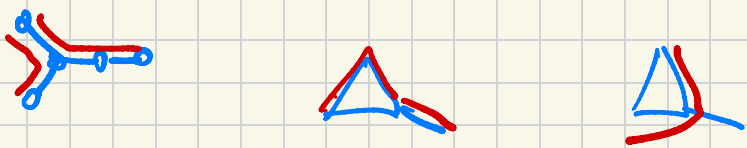
1 push R }

1 pop R ← dequeue

invariant:

$e = 3l + R$

16.32 maximal vs maximum path in conn graph



16.48 sp tree or "disconnected" BFS

16.11 strong connectedness of digraph in time

BFS
reverse
BFS