1. A random graph on \( n \) vertices can be obtained in the following way. The vertex set of the graph is \( \{1, \ldots, n\} \) and we draw all the possible (undirected) edges independently with probability \( 1/2 \) (in other words, for all the \( \binom{n}{2} \) possible edges toss a coin and draw the edge if you got heads). The same way you can define a random graph on countably many vertices. Show that there is only ONE countable random graph: that is, almost all random graphs on a countable set are isomorphic.

2. Is there a measurable subset \( A \subseteq [0, 1] \) such that for every nonempty open interval \( I \subseteq [0, 1] \) both \( I \cap A \) and \( I \setminus A \) have positive measure?

3. Does there exist continuously many subsets of \( [0, 1] \), all of measure 1/2, such that any two different sets have intersection of measure 1/4?

4. Can you cover the set of nonnegative integers with finitely many disjoint arithmetic progressions of distinct differences?

5. Let \( F = F_2 \) be the free group on two generators. Show that for every \( g \in F \) the intersection of finite index subgroups of \( F \) containing \( g \) equals the cyclic group generated by \( g \).

6. For subsets \( A, B \subseteq \mathbb{Z} \) let \( A + B = \{ a + b \mid a \in A, b \in B \} \). For \( A \subseteq \mathbb{N} \) let us define the upper density as

\[
\overline{d}(A) = \limsup_{n \to \infty} \frac{|A \cap [1, n]|}{n}
\]

Is there a subset \( A \subseteq \mathbb{N} \) of zero upper density such that \( A + A = \mathbb{N} \)?

7. Are there 4 points on the plane such that all 6 possible distances are odd integers?

8. Show that \( \text{PSL}(2, \mathbb{Z}) \cong C_2 \ast C_3 \).

9. Prove that every large enough alternating group can be generated by an element of order 2 and an element of order 3.

10. Show that a 3-regular graph on \( n \) vertices has at most \( 2n/2 + 1 \) cycles.

11. Prove that the real function \( f(x) = x^2 \) can be obtained as the sum of three periodic real functions.
12. Assume that \( a_n \) and \( b_n \) are positive non-decreasing sequences such that both \( \sum_n 1/a_n \) and \( \sum_n 1/b_n \) are divergent. Does this imply that
\[
\sum_n \frac{1}{a_n + b_n}
\]
is divergent?

13. Are there two proper subsets \( A, B \) of the plane such that for any \( A' \sim A \) and \( B' \sim B \) we have \(|A' \cap B'| = 2005|\)?

14. Show that if \( A \) is any set of points of size \( d + 2 \) in \( R^d \) then there exists a partition \( A = B \cup C \) such that the convex hull of \( B \) intersects the convex hull of \( C \).

15. Let \( G \) be a finitely generated group. Show that for every integer \( n \) there are finitely many subgroups of \( G \) of index \( n \).

16. Is there a continuous function on \([0,1]\) which takes on every value in its range countably infinitely many times?