Due **Thurs.**

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$\boxed{\text{HW}}$ Prove that the chromatic polynomial is a polynomial.

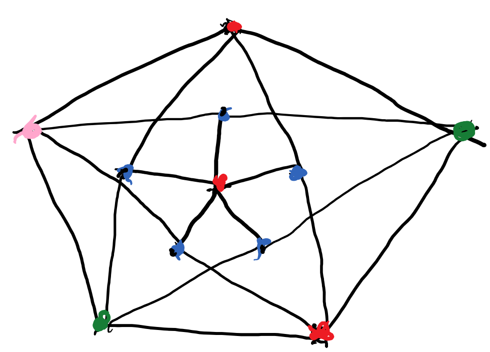
(Reminder: for graph G, chromatic poly. $f_G(t)$ is defined

$f_G(t) = \#$ of legal colorings of G with t colors

for $t = \mathbb{N}$.

(Note: tree problem from yesterday should be $t(t-1)^{n-1}$ – my bad)

Draw a graph with no 3-cycles that is not 3-colorable.



M

$\chi(G) \le$ max deg. of any vertex + 1

Induction on n, # of vertices.

Base case ($n = 1$): trivial.

Suppose true for $n-1$ vertices

If G has n vertices, then consider a subgraph without some vertex. This is colorable by inductive hypothesis. Transfer this coloring to the full graph. Because the last has degree $\le M$, so there is at least one unused color that can be used

What about an algorithmic approach?

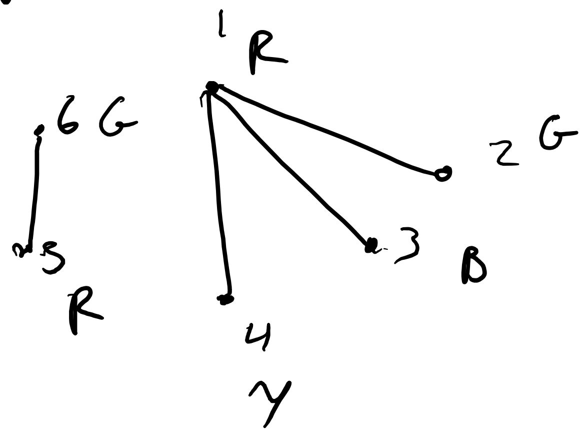
Index vertices.

Color first vertex w/ adj. If colored vertices diff colors.

If legal, first existing color.

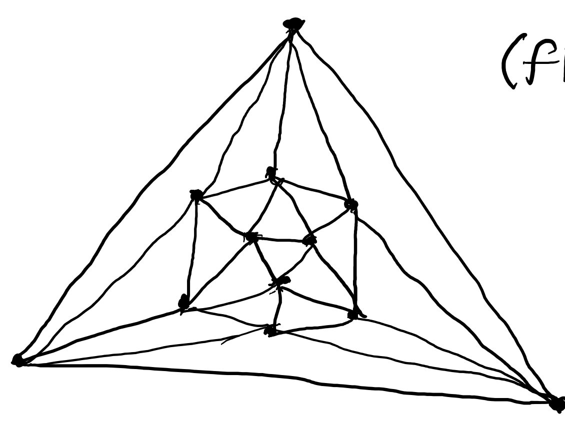
Else new color. Else

Color first remaining vertex first color.



Draw a famous regular planar graph of degree 5.

(flattened)



icosahedron
(20 faces - all triangles)

6-color Theorem

Induction on n

$n \leq 6 \rightarrow$ obviously 6-colorable (all diff colors)

Assume planar graphs with $n = k-1$ nodes colorable.

Consider planar graph G with $n = k$.

By lemma, G must have node with deg ≤ 5.

Let that node be i. Consider subgraph w/o i.

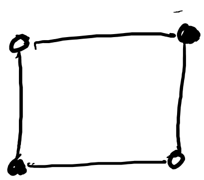
H planar (removing vertices/edges cannot make it nonplanar)

H 6-colorable by inductive hypothesis.

Add back in - connected to at most 5 colors - at least 1 available - 6-colorable.

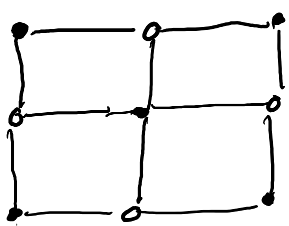
Max independence # on $n \times n$ grid is realized by checkerboard pattern

For an $n \times n$ grid, $\alpha(G) = \lceil \frac{n^2}{2} \rceil$



$\lceil \frac{2^2}{2} \rceil = 2$

LB: counting proof.

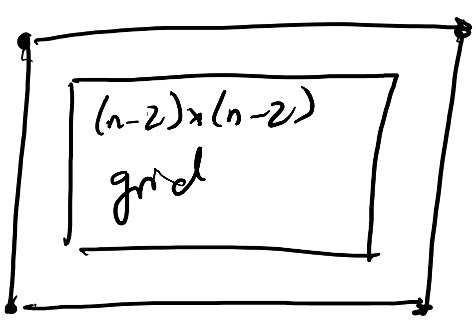


$\lceil \frac{3^2}{2} \rceil = \frac{10}{2} = 5$

For any $1 \le k \le n-1$ grid, $\alpha(G) = \lceil \frac{k^2}{2} \rceil$

even: $\frac{n^2}{2} = \lceil \frac{n^2}{2} \rceil$

odd: $\frac{n^2+1}{2} = \lceil \frac{n^2}{2} \rceil$



4(n-1) cycle

2(n-1)

if odd

$$\left\lceil \frac{(n-2)^2}{2} \right\rceil + \frac{4n-4}{2} \le \frac{(n^2 \cancel{-4n} \cancel{+4}) + 1 \cancel{+4n} \cancel{-4}}{2}$$

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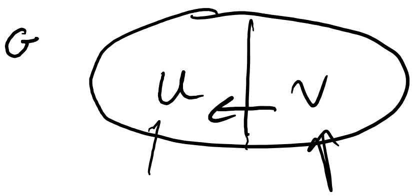
show $\alpha(G) \le \frac{n}{2}$



U is independent

k-regular.

$|U|k$ $\qquad$ $\frac{kn}{2} - k|U|$

$|U| = \alpha > \frac{n}{2} \implies \frac{kn}{2} - k|U| < 0$ (bad - contradiction)

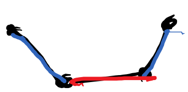
How to realize bound?

Complete bipartite graph w/ equal parts where each part has cardinality of k.

Let M be a maximal matching of a graph G. Show $\nu(G) \le 2|M|$. $\quad$ *can't add vertex more than once - matching

$\nu(G) = |M^*|$

M - maximal. $\quad$ - $G_M(V, E) \to |E| = |M|$



(there is a graph s.t $\frac{\nu(G)}{|M|} \le 2$)

$G_{M^*} = (V^*, E^*)$ $\qquad$ $|E^*| = |M^*|$

If $(i, j) \in E^*$ one of i, j should be in N (otherwise non-maximal)

$\underline{2|E|} = |V| \ge |E^*| = \underline{\nu(G)}$.

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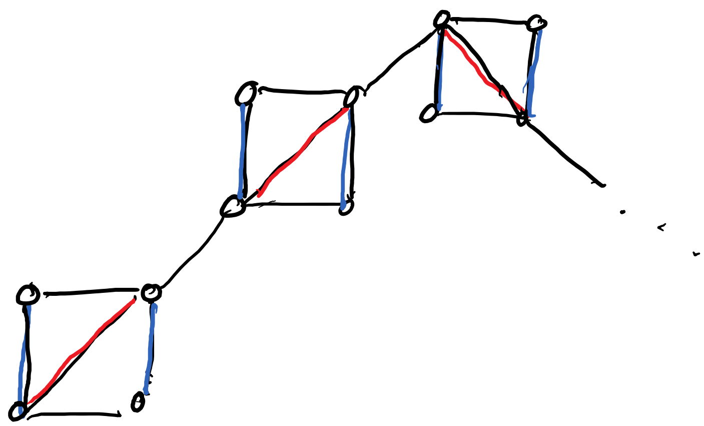
$(\forall k)(\exists G)(\nu(G) = 2|M| = 2k$ + G is connected)



— maximum

— maximal