

### Supplementary Problems for Problem Set 9

The first two relate to the conditions (7.6) of the text on the flow  $\phi(t, x)$  of a dynamical system.

1. Solve the one-dimensional system

$$\dot{x} = -x^2$$

explicitly for  $\phi$  and verify condition (iii) of (7.6).

2. Suppose you are given a smooth function  $\phi$  satisfying the conditions (7.6). Show that there is a function  $f(y)$  of  $y$  only – and find it! – such that  $\phi(t, x)$  is the solution of the initial-value problem

$$\dot{y} = f(y), \quad y(0) = x.$$

3. First do problem 2 of Problem Set 8.3.1. Then consider the nonlinear system

$$\dot{x} = y, \quad \dot{y} = -x^3,$$

which has the same linearization about the origin and answer the same question: is the origin stable or unstable?

4. As in the preceding problem, but for the system

$$\dot{x} = y - x^2, \quad \dot{y} = 0.$$