

Supplementary Problem for Problem Set 8

Supplementary problem: find the volume evolution in phase space for the following systems:

1. The Lorenz system

$$\dot{x} = \sigma(y - x), \quad \dot{y} = rx - y - xz, \quad \dot{z} = xy - bz,$$

where σ, r, b are positive constants.

2. Euler's equations of rigid-body dynamics

$$\dot{w}_1 = w_2 w_3, \quad \dot{w}_2 = -w_3 w_1, \quad \dot{w}_3 = -\mu w_1 w_2,$$

where μ is a constant.

3. Hamilton's canonical equations

$$\dot{x}_k = \frac{\partial H}{\partial x_{n+k}}, \quad \dot{x}_{n+k} = -\frac{\partial H}{\partial x_k}, \quad k = 1, 2, \dots, n,$$

where $H = H(x)$ is a smooth function of the $2n$ variables $x = (x_1, \dots, x_{2n})$.