

## Supplementary Problems for Chapter 5

1. Consider the third-order equation having a regular singular point at the origin:

$$z^3 w''' + a_1(z)z^2 w'' + a_2(z)zw' + a_1(z)w = 0,$$

where  $a_1, a_2, a_3$  are analytic in a domain including the origin. This can be transformed to a system of three first-order equations, by the definitions  $w_1 = w, w_2 = zw'_1, w_3 = zw'_2$ , of the form  $zW' = A(z)W$  where  $W$  is the vector with components  $w_1, w_2, w_3$  and  $A$  is a three-by-three matrix. Work out  $A$ .

2. Consider the equation

$$(1 - z)z^2 w'' + (z - 4)zw' + 6w = 0.$$

- (a) Verify that this equation has a regular singular point at the origin.
- (b) Find the indicial equation and the indices relative to this point.
- (c) For the index with the greater real part, find the recursion relation for the coefficients in the series solution.

The following two problems relate to singular points at infinity. These are investigated by making the transformation  $t = 1/z$  and investigating the singular points at  $t = 0$ . In each case determine whether the point in question is a point of analyticity, a regular singular point, or an irregular singular point. In the case of a regular singular point, find the indices.

3. The equation  $w'' + w = 0$ .
4. The equation  $z^2 w'' + w = 0$