

Alexander A. Razborov

Personal data

Born on Feb 16, 1963 in the town Belovo, USSR.

Education

- 1985-87: graduate student at the Steklov Mathematical Institute, Moscow. Advisor: Prof. S.I.Adian.
- 1980-85: undergraduate student of the Moscow University, Department of Mechanics and Mathematics.

Degrees

- 1991: doctoral thesis, *Lower bounds in the Boolean Complexity*.
- 1987: PhD, *On systems of equations in free groups*.

Employment

- 2008-current: Andrew MacLeish Distinguished Service Professor, University of Chicago.
- 2000-2008: Principal Researcher, Steklov Mathematical Institute.
- 1991-2000: Leading Researcher, Steklov Mathematical Institute.
- 1987-1991: Researcher, Steklov Mathematical Institute.

Part-time positions

- 2008-current: Principal Researcher, Steklov Mathematical Institute.
- 2008-current: Resident Adjoint Professor, Toyota Technological Institute at Chicago.

Visiting positions

- 2003-2008: Visiting Professor, Institute for Advanced Study, Princeton

- 2000-2003: Member, Institute for Advanced Study, Princeton
- 1999-2000: Visiting Researcher, Department of Computer Science, Princeton University
- 1993-1994: Member, Institute for Advanced Study, Princeton

Honors

- 2013: David P. Robbins Prize of the American Mathematical Society.
- 2007: Göedel Prize of the European Association for Theoretical Computer Science and the Special Interest Group on Algorithms and Computation Theory of the Association for Computing Machinery.
- 1990: Rolf Nevanlinna Prize of the International Mathematical Union.

Membership

- 2020: American Academy of Arts and Sciences
- 2000: corresponding member of the Russian Academy of Science
- 1993: Academia Europea

Editorial Boards

- Forum of Mathematics (Π and Σ).
- Sbornik: Mathematics
- Combinatorica.
- Computational Complexity.
- Combinatorics, Probability and Computing.
- Moscow Journal of Combinatorics and Number Theory
- Lobachevskii Journal of Mathematics

Steering Committee

- Computer Science Symposium in Russia, 2012-current

Program Committees

- Computational Complexity Conference 2021
- International Conference “Computer Science Symposium in Russia” 2021
- Oberwolfach workshop “Proof Complexity and Beyond” 2017
- Innovations in Theoretical Computer Science Conference 2017
- Association for Symbolic Logic North American Annual Meeting 2015
- International Conference “Computer Science Symposium in Russia” 2015
- International Conference “Computer Science Symposium in Russia” 2014
- International Colloquium on Automata, Languages and Programming 2013
- International Conference “Computer Science Symposium in Russia” 2013
- International Conference “Computer Science Symposium in Russia” 2012
- International Conference “Computer Science Symposium in Russia” 2011
- IEEE Conference on Computational Complexity 2009
- International Conference on Logic for Programming, Artificial intelligence, and Reasoning 2008
- International Conference “Computer Science Symposium in Russia” 2008 **chair of Theory Track**

- Symposium on Foundations of Computer Science 2007
- International Conference “Computer Science Symposium in Russia” 2007
- Workshop on Logic, Language, Information and Computation 2007
- 13th International Congress of Logic, Methodology and Philosophy of Science 2007
- International Conference “Computer Science Symposium in Russia” 2006
- 2nd International Conference “Methods of logic in mathematics” 2005
- International Conference Dedicated to the 100th Anniversary of P.S.Novikov 2001
- Workshop on Circuit and Proof Complexity 2001
- Workshop on Complexity of Proofs and Computations 2000
- Logic Colloquium 2000
- Computational Complexity 2000
- Computer Science Logic 1998
- Workshop on Complexity Theory 1997
- Random97 conference
- Computer Science Logic 1994

Current students

- L. Coregliano
- S. Pang
- T. Papamakarios
- D. Quintana

Former students

- A. Nogin.
- O. Verbitsky (jointly with S. I. Adian).
- M. Alekhnovich (undergraduate)
- V. Podolski (undergraduate)
- R. Kulkarni (jointly with J. Simon)
- Y. Li
- P. Hatami

Plenary and invited talks

- 8th European Congress of Mathematicians, Portoroz, 2020 (re-scheduled for 2021)
- 14th Latin American Theoretical Informatics Symposium, Sao Paolo, 2020 (January 2021, online), *Theons and Quasirandomness*
- Association for Symbolic Logic North American Annual Meeting, Irvine, 2020 (cancelled)
- Workshop on Proof Complexity, Banff, Canada, 2020, *Hardness Condensation*
- Steklov International Mathematical Center, inaugural conference, Moscow, 2019, *Limits of Combinatorial Objects*
- Symposium on 50 Years of Complexity theory in Honor of Stephen Cook, Toronto, 2019, *P, NP and Proof Complexity*
- ICERM workshop on Real Algebraic Geometry and Optimization, Providence, 2018, *The Tale of One Variety (And Its Family)*.
- Clay Institute workshop on Complexity Theory, Oxford, 2018, *Grand Challenges in Complexity Theory through the Lens of Proof Theory*

- Building Bridges II Conference, Budapest, 2018, *Semantic Limits of Combinatorial Objects*
- VIII RASA-USA conference, Chicago, 2017, *Solving Satisfiability*
- Avi is 60 Conference, Princeton, 2016, *Propositional Proof Complexity: Fifteen (or so) Years After*
- Computability in Europe, Bucharest, 2015, *Complexity of Propositional Proofs*
- Stockholm Mathematics Centre Colloquium, 2015, *Continuous Combinatorics*
- Crystals, Quasicrystals and Random Networks Workshop (ICERM), Providence, 2015, *Syntactic Methods in Continuous Combinatorics*
- Probabilistic and Extremal Combinatorics Workshop (IMA), Minneapolis, 2014, *Flag Algebras: an Interim Report*
- Mathematics in Armenia: Advances and Perspectives, Tsaghkadzor, 2013, *Flag Algebras*
- EXCILL2: Extremal Combinatorics at Illinois (plenary lecture), 2013, *Flag Algebras: an Interim Report*
- Indiana University Turing Symposium (keynote lecture), 2012, *Flag Algebras*
- 8th Annual Graduate Student Combinatorics Conference (keynote lecture), Urbana-Champaign, 2012, *Topics in Extremal Combinatorics*
- Saint Petersburg Mathematical Society, 2011, *Flag Algebras*
- Aisenstadt Lectures, Montreal, 2010 (various topics)
- 5th International Computer Science Symposium in Russia (opening lecture), Kazan, 2010, *Complexity of Propositional Proofs*
- Twenty-first annual Gödel Lecture, Association for Symbolic Logic North American Annual Meeting, Washington DC, 2010

- Barriers in Complexity Theory Workshop, Princeton, 2009, *Somewhat Impartial Survey of Results in Circuit Complexity*
- Association for Symbolic Logic North American Annual Meeting, Notre Dame University, 2009, *Flag Algebras*
- Midwest Model Theory Meeting, Notre Dame University, 2008, *Flag Algebras*
- China Theory Week, Beijing, 2007, *Complexity of Propositional Proofs*
- Workshop on Computational, Descriptive and Proof Complexity, and Algorithms, Moscow, 2007, *Grand Challenges of Proof Complexity*
- Symposium on Logical Foundations of Computer Science, New York, 2007, *Natural Proofs: Ten Years After*
- Pacific Institute for the Mathematical Sciences, 10th Anniversary Lecture, Vancouver, 2006, *Feasible Proofs and Computations*
- European Mathematical Society weekend, Prague, 2004, *Feasible Proofs and Computations.*
- Joint meeting of 31st International Colloquium on Automata, Languages and Programming and 19th Annual IEEE Symposium on Logic in Computer Science, Turku, 2004, *Feasible Proofs and Computations: Partnership and Fusion.*
- International Workshop on Logic and Complexity in Computer Science Dedicated to the 60th Anniversary of A. Slissenko, Paris, 2001, *Proof Complexity of pigeonhole principles.*
- 5th Developments of Language Theory Conference, Vienna, 2001, *Propositional Proof Complexity.*
- International Conference Dedicated to the 100th Anniversary of P.S.Novikov, Moscow, 2001, *Complexity of Propositional Proofs.*
- Alfred Tarsky lectures, Berkeley, 2000.
- Vision in Mathematics, Tel-Aviv, 1999, *Complexity of Proofs and Computations.*

- Collegium Logicum, Vienna, 1998, *Complexity of Resolution Proofs*.
- Logic Colloquium, Prague, 1998, *Complexity of Resolution Proofs*.
- Proof Theory and Complexity, Aarhus, 1998, *Lower Bounds for Algebraic Proof Systems*.
- Paul Erdős Lectures, Jerusalem, 1998.
- Coxeter Lectures, Fields Institute, Toronto, 1998.
- 23rd International Colloquium on Automata, Languages and Programming, Paderborn, 1996, *Lower bounds for propositional proofs and independence results in Bounded Arithmetic*.
- DIMACS Distinguished Lectures Series, Rutgers University, 1996, *Lower bounds for propositional proofs and independence results in Bounded Arithmetic*.
- Mathematical Foundations of Computer Science, Prague, 1995, *Lower Bounds for Propositional Proofs and Independence Results in Bounded Arithmetic*.
- The seventh International Conference on Random Structures and Algorithms, Atlanta, 1995, *Pseudorandom Function Generators in Proof Theory and Complexity Theory*.
- Workshop on Geometrical and Combinatorial Methods in the Group Theory, Edinburgh, 1993, *On systems of equations in free groups*.
- SWAT 92, Helsinki, 1992, *On small depth threshold circuits*.
- Feasible Mathematics II, Cornell University, 1992, *Bounded arithmetic and lower bounds in Boolean complexity*.
- 8th Fundamentals of Complexity Theory, Gosen, 1991, *Lower bounds for deterministic and nondeterministic branching programs*.
- International Congress of Mathematicians at Berkeley, 1986, *Lower Bounds for Monotone Complexity of Boolean Functions*.

Journal Publications

- [1] A. A. Razborov. On systems of equations in a free group. *Izvestiya AN SSSR, ser. matem.*, 48(4):779–832, 1984. English Translation in *Math. USSR Izvestiya*, 25(1):115–162, 1985.
- [2] A. A. Razborov. Lower bounds for the monotone complexity of some boolean functions. *Doklady Akademii Nauk SSSR*, 281(4):798–801, 1985. English Translation in *Soviet Math. Dokl.*, 31:354–357, 1985.
- [3] A. A. Razborov. Lower bounds of monotone complexity of the logical permanent function. *Matematicheskie Zametki*, 37(6):887–900, 1985. English Translation in *Mathem. Notes of the Academy of Sci. of the USSR*, 37:485–493, 1985.
- [4] A. Razborov. Lower bounds on the size of bounded-depth networks over a complete basis with logical addition. *Mathematical Notes of the Academy of Sciences of the USSR*, 41(4):598–607, 1987. English translation in 41:4, pages 333–338.
- [5] S. I. Adian and A. A. Razborov. Periodical groups and Li algebras. *Russian Mathematical Surveys*, 42(2):3–68, 1987.
- [6] A. Razborov. The gap between the chromatic number of a graph and the rank of its adjacency matrix is superlinear. *Discrete Mathematics*, 108:393–396, 1992.
- [7] A. Razborov. On the distributional complexity of disjointness. *Theoretical Computer Science*, 106:385–390, 1992.
- [8] A. Razborov. Kolmogorov and the complexity of algorithms. *Bull. London. Math. Soc.*, 22:79–82, 1990.
- [9] A. Razborov. Lower bounds on the size of switching-and-rectifier networks for symmetric Boolean functions. *Mathematical Notes of the Academy of Sciences of the USSR*, 48(6):79–91, 1990.
- [10] A. Razborov. Applications of matrix methods to the theory of lower bounds in computational complexity. *Combinatorica*, 10(1):81–93, 1990.
- [11] M.S. Paterson and A. A. Razborov. The set of minimal braids is *co-NP*-complete. *Journal of Algorithms*, 12:393–408, 1991.

- [12] A. Borodin, A. Razborov, and R. Smolensky. On lower bounds for read- k times branching programs. *Computational Complexity*, 3(1):1–18, 1993.
- [13] M. Goldmann, J. Hastad, and A. Razborov. Majority gates vs. general weighted threshold gates. *Computational Complexity*, 2:277–300, 1992.
- [14] J. Hastad, A. Razborov, and A. Yao. On the shrinkage exponent for read-once formulae. *Theoretical Computer Science*, 141:269–282, 1995.
- [15] A. Razborov and A. Wigderson. $n^{\Omega(\log n)}$ lower bounds on the size of depth 3 threshold circuits with AND gates at the bottom. *Information Processing Letters*, 45:303–307, 1993.
- [16] A. Razborov, E. Szemerédi, and A. Wigderson. Constructing small sets that are uniform in arithmetic progressions. *Combinatorics, Probability and Computing*, 2:513–518, 1993.
- [17] A. Razborov. On the parameterization of solutions for equations in free groups. *International Journal of Algebra and Computation*, 3(3):251–273, 1993.
- [18] A. Razborov. Unprovability of lower bounds on circuit size in certain fragments of Bounded Arithmetic. *Izvestiya of the RAN*, 59(1):201–222, 1995. See also *Izvestiya: Mathematics* 59:1, 205–227.
- [19] A. Razborov and S. Rudich. Natural proofs. *Journal of Computer and System Sciences*, 55(1):24–35, 1997.
- [20] S. Buss, R. Impagliazzo, J. Krajíček, P. Pudlák, A. Razborov, and J. Sgall. Proof complexity in algebraic systems and bounded depth Frege systems with modular counting. *Computational Complexity*, 6(3):256–298, 1996/1997.
- [21] S. Jukna and A. Razborov. Neither reading few bits twice nor reading illegally helps much. *Discrete Applied Mathematics*, 85(3):223–238, 1998.
- [22] A. Razborov, A. Wigderson, and A. Yao. Read-once branching programs, rectangular proofs of the pigeonhole principle and the transversal calculus. *Combinatorica*, 22(4):555–574, 2002.
- [23] A. Razborov. Lower bounds for the polynomial calculus. *Computational Complexity*, 7:291–324, 1998.

- [24] B. Kashin and A. Razborov. Improved lower bounds on the rigidity of Hadamard matrices. *Matematicheskie Zametki*, 63(4):535–540, 1998.
- [25] S. Jukna, A. Razborov, P. Savicky, and I. Wegener. On P versus $NP \cap co - NP$ for decision trees and read-once branching programs. *Computational Complexity*, 8(4):357–370, 1999.
- [26] D. Grigoriev and A. Razborov. Exponential complexity lower bounds for depth 3 arithmetic circuits in algebras of functions over finite fields. *Applicable Algebra in Engineering, Communication and Computing*, 10(6):465–487, 2000.
- [27] M. Alekhnovich, E. Ben-Sasson, A. Razborov, and A. Wigderson. Space complexity in propositional calculus. *SIAM Journal on Computing*, 31(4):1184–1211, 2002.
- [28] M. Alekhnovich, E. Ben-Sasson, A. Razborov, and A. Wigderson. Pseudorandom generators in propositional proof complexity. *SIAM Journal on Computing*, 34(1):67–88, 2004.
- [29] M. Alekhnovich and A. Razborov. Lower bounds for the polynomial calculus: non-binomial case. *Proceedings of the Steklov Institute of Mathematics*, 242:18–35, 2003.
- [30] A. Razborov. Resolution lower bounds for the weak functional pigeon-hole principle. *Theoretical Computer Science*, 303(1):233–243, 2003.
- [31] A. Razborov. Resolution lower bounds for perfect matching principles. *Journal of Computer and System Sciences*, 69(1):3–27, 2004.
- [32] A. Razborov. Quantum communication complexity of symmetric predicates. *Izvestiya of the Russian Academy of Science, Mathematics*, 67(1):159–176, 2003.
- [33] A. Razborov. An upper bound on the threshold quantum decoherence rate. *Quantum Computation and Information*, 4(3):222–228, 2004.
- [34] A. Razborov. Guessing more secrets via list decoding. *Internet Mathematics*, 2(1):21–30, 2005.
- [35] V. Lifschitz and A. Razborov. Why are there so many loop formulas? *ACM Transactions on Computational Logic*, 7(2):261–268, 2006.

- [36] A. Razborov. Flag algebras. *Journal of Symbolic Logic*, 72(4):1239–1282, 2007.
- [37] A. Razborov and S. Yekhanin. An $\Omega(n^{1/3})$ lower bound for bilinear group based private information retrieval. *Theory of Computing*, 3:221–238, 2007.
- [38] M. Alekhnovich and A. Razborov. Resolution is not automatizable unless $W[P]$ is tractable. *SIAM Journal on Computing*, 38(4):1347–1363, 2008.
- [39] A. Razborov. On the minimal density of triangles in graphs. *Combinatorics, Probability and Computing*, 17(4):603–618, 2008.
- [40] A. Razborov. A simple proof of Bazzi’s theorem. *ACM Transactions on Computation Theory*, 1(1), 2009.
- [41] V. Guruswami, J. Lee, and A. Razborov. Almost euclidean subspaces of ℓ_1^N via expander codes. *Combinatorica*, 30(1):47–68, 2010.
- [42] A. Razborov and A. Sherstov. The sign-rank of AC^0 . *SIAM Journal on Computing*, 39(5):1833–1855, 2010.
- [43] A. Razborov. On 3-hypergraphs with forbidden 4-vertex configurations. *SIAM Journal on Discrete Mathematics*, 24(3):946–963, 2010.
- [44] F. Eisenbrand, N. Hähnle, A. Razborov, and T. Rothvoß. Diameter of polyhedra: Limits of abstraction. *Mathematics of Operations Research*, 35:786–794, 2010.
- [45] A. Razborov. On the Fon-der-Flaass interpretation of extremal examples for Turan’s (3,4)-problem. *Proceedings of the Steklov Institute of Mathematics*, 274:247–266, 2011.
- [46] M. Alekhnovich and A. Razborov. Satisfiability, branch-width and Tseitin tautologies. *Computational Complexity*, 20:649–678, 2011.
- [47] O. Beyersdorff, N. Galesi, M. Lauria, and A. Razborov. Parameterized bounded-depth Frege is not optimal. *ACM Transactions on Computation Theory*, 7, article 7, 2012.

- [48] Hatami H, J. Hladky, D. Kral, S. Norin, and A. Razborov. Non-three-colorable common graphs exist. *Combinatorics, Probability and Computing*, 21(5):734–742, 2012.
- [49] Hatami H, J. Hladky, D. Kral, S. Norin, and A. Razborov. On the number of pentagons in triangle-free graphs. *Journal of Combinatorial Series, ser. A*, 120(3):722–732, 2013.
- [50] A. Razborov. On the Caccetta-Haggkvist conjecture with forbidden subgraphs. *Journal of Graph Theory*, 74(2):236–248, 2013.
- [51] A. Razborov. On Turan’s (3,4)-problem with forbidden configurations. *Matematicheskie Zametki*, 95(2):271–281, 2014.
- [52] A. Razborov and E. Viola. Real advantage. *ACM Transactions on Computation Theory*, 5:article 7, 2013.
- [53] A. Razborov. A product theorem in free groups. *Annals of Mathematics*, 179(2):405–429, 2014.
- [54] A. Razborov. Pseudorandom generators hard for k -DNF resolution and polynomial calculus resolution. *Annals of Mathematics*, 181(3):415–472, 2015.
- [55] A. Razborov. A new kind of tradeoffs in propositional proof complexity. *Journal of the ACM*, 62(3):article 16, 2016.
- [56] Y. Li, A. Razborov, and B. Rossman. On the AC^0 complexity of subgraph isomorphism. *SIAM Journal on Computing*, 46(3):936–971, 2017.
- [57] L. Coregliano and A. Razborov. On the density of transitive tournaments. *Journal of Graph Theory*, 85(1):12–21, 2017.
- [58] A. Razborov. On the width of semi-algebraic proofs and algorithms. *Mathematics of Operation Research*, 42(4):1106–1134, 2017.
- [59] O. Pikhurko and A. Razborov. Asymptotic structure of graphs with the minimum number of triangles. *Combinatorics, Probability and Computing*, 26(1):138–160, 2017.
- [60] A. Razborov. On space and depth in resolution. *Computational Complexity*, 27(3):511–559, 2018.

Conference Proceedings, Collections etc.: original results not (yet) published in journals

- [1] A. A. Razborov. Bounded-depth formulae over $\{\wedge, \oplus\}$ and some combinatorial problems. In S. I. Adian, editor, *Problems of Cybernetics. Complexity Theory and Applied Mathematical Logic*, pages 149–166. VINITI, Moscow, 1988. In Russian.
- [2] A. Razborov. On the method of approximation. In *Proceedings of the 21st ACM Symposium on Theory of Computing*, pages 167–176, 1989.
- [3] A. Razborov. On submodular complexity measures. In M. S. Paterson, editor, *Boolean Function Complexity. London Math. Soc., Lecture Note Series* 169, pages 76–83. Cambridge University Press, 1992.
- [4] A. Razborov. An equivalence between second order bounded domain bounded arithmetic and first order bounded arithmetic. In P. Clote and J. Krajíček, editors, *Arithmetic, Proof Theory and Computational Complexity*, pages 247–277. Oxford University Press, 1992.
- [5] A. Razborov. Bounded Arithmetic and lower bounds in Boolean complexity. In P. Clote and J. Remmel, editors, *Feasible Mathematics II. Progress in Computer Science and Applied Logic, vol. 13*, pages 344–386. Birkhäuser, 1995.
- [6] J. Nordström and A. Razborov. On minimal unsatisfiability and time-space trade-offs for k-DNF resolution. In *Proceedings of the 38th International Colloquium on Automata, Languages and Programming*, pages 642–653, 2011.
- [7] A. Atserias, I. Bonacina, S. Rezende, M. Lauria, J. Nordström, and A. Razborov. Clique is hard on average for regular resolution. In *Proceedings of the 50th ACM Symposium on the Theory of Computing*, pages 866–877, 2018.

Preprints, Submitted Manuscripts etc.

- [1] A. A. Razborov. On rigid matrices. Manuscript in Russian, June 1989.

- [2] A. Razborov. On provably disjoint **NP**-pairs. Technical Report RS-94-36, Basic Research in Computer Science Center, Aarhus, Denmark, 1994. Available at <http://www.brics.aau.dk/RS/94/36/BRICS-RS-94-36.ps.gz>.
- [3] A. Razborov. Improved resolution lower bounds for the weak pigeon-hole principle. Technical Report TR01-055, Electronic Colloquium on Computational Complexity, 2001.
- [4] D. Mubayi and A. Razborov. Polynomial to exponential transition in Ramsey theory. Technical Report 1901.06029 [math.CO], arXiv e-print, 2019, to appear in *Proceedings of the London Mathematical Society*.
- [5] L. Coregliano and A. Razborov. Semantic limits of dense combinatorial objects. Technical Report 1910.08797 [math.CO], arXiv e-print, 2019, to appear in *Russian Mathematical Surveys*.
- [6] N. Mull, S. Pang, and A. Razborov. On CDCL-based proof systems with the ordered decision strategy. Technical Report 1909.04135 [cs.LO], arXiv e-print, 2019.

Conference Proceedings: survey talks

- [1] A. A. Razborov. Lower bounds for monotone complexity of boolean functions. In *Proceedings of the International Congress of Mathematicians*, volume 2, pages 1478–1487, Berkeley, California, USA, 1986. In Russian. For the English translation see *Amer. Math. Soc. Transl.*, 147(2):75-84, 1990.
- [2] S. I. Adian, A.A. Razborov, and N.N Repin. Upper and lower bounds for nilpotency classes of Lie algebras with Engel conditions. In *Group Theory, Proceedings of the Singapore Group Theory Conference held at the National University of Singapore, June 8-19,1987*, pages 57–75. Walter de Gruyter, 1989.
- [3] A. Razborov. Lower bounds for deterministic and nondeterministic branching programs. In *Proceedings of the 8th FCT, Lecture Notes in Computer Science*, 529, pages 47–60, New York/Berlin, 1991. Springer-Verlag.

- [4] A. Razborov. On small depth threshold circuits. In *Proceedings of the SWAT 92, Lecture Notes in Computer Science*, 621, pages 42–52, New York/Berlin, 1992. Springer-Verlag.
- [5] A. Razborov. On systems of equations in free groups. In *Combinatorial and Geometric Group Theory, Edinburgh 1993. London Mathematical Society Lecture Note Series*, 204, pages 269–283. Cambridge University Press, 1995.
- [6] A. Razborov. Lower bounds for propositional proofs and independence results in Bounded Arithmetic. In F. Meyer auf der Heide and B. Monien, editors, *Proceedings of the 23rd ICALP, Lecture Notes in Computer Science*, 1099, pages 48–62, New York/Berlin, 1996. Springer-Verlag.
- [7] A. Razborov. Proof complexity of pigeonhole principles. In *Proceedings of the 5th DLT, Lecture Notes in Computer Science*, 2295, pages 100–116, New York/Berlin, 2002. Springer-Verlag.
- [8] A. Razborov. Propositional proof complexity. *Journal of the ACM*, 50(1):80–82, 2003.
- [9] A. Razborov. Feasible proofs and computations: partnership and fusion. In *Proceedings of the 31st International Colloquium, Lecture Notes in Computer Science*, 3142, pages 8–14, New York/Berlin, 2004. Springer-Verlag. Also appeared in Proceedings of the 19th LICS conference.