Complexity Theory

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You may (and are mildly encouraged to) work together on solving homework problems, but please put all the names of your collaborators at the top of the assignment. Everyone must turn in his/her own independently written solution.

Shopping for solutions on the Internet is strongly discouraged. If you encounter it anyway, you must completely understand the proof, explain it in your own words and include the URL.

PDF file **prepared from a TeX source** is the preferred format. In that case you will get back your feedback in equally neat form.

Homework 2, due May 7

1. Determine the complexity of the following problem: given a CNF ϕ in which every clause is either completely negative or completely positive, decide if ϕ is satisfiable.

2. (a) Prove that $\mathsf{P}^{\mathsf{NP} \cap co-\mathsf{NP}} = \mathsf{NP} \cap co - \mathsf{NP}$ or, in other words, that $\mathsf{NP} \cap co - \mathsf{NP}$ is closed under *Cook* reductions.

   (b) Prove that $\mathsf{NP}^{\mathsf{NP} \cap co-\mathsf{NP}} = \mathsf{NP}$ or, in other words, this class of oracles does not add anything new to non-deterministic computations.

3. Reingold's celebrated result mentioned in class can be stated as follows: the connectivity problem in *undirected* graphs is in L.

   Prove that testing if a given undirected graph is bi-partite is also in L.

4. Recall that $\mathbf{R}$ consists of all languages L for which there exists a polynomial time non-deterministic Turing machine M with the following acceptance conditions:

$$\begin{cases} x \in L \Longrightarrow \mathbf{P}[M \text{ accepts } x] \geq 1/2 \\ x \notin L \Longrightarrow \mathbf{P}[M \text{ rejects } x] = 1. \end{cases}$$

   Prove that $\mathbf{R}^{\mathbf{BPP}} = \mathbf{BPP}^{\mathbf{R}}$.

5. Let us call a language $L \subseteq \{0,1\}^*$ *diadic* if for every $x \in L$ and every $i \leq |x|$ with $x_i = 1$, i is a power of two.

   Prove that the class of languages that can be recognized by a polynomial time oracle machine with a diadic oracle is equal to P/poly.