Complexity Theory

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You may (and are mildly encouraged to) work together on solving homework problems, but please put all the names of your collaborators at the top of the assignment. Everyone must turn in his/her own independently written solution.

You (obviously) have to prove all your answers, and everything that was stated in class can be used without a proof unless explicitly forbidden in the statement.

Shopping for solutions on the Internet is strongly discouraged. If you encounter it anyway, you must completely understand the proof, explain it in your own words and include the URL.

PDF file **prepared from a TeX source** is the preferred format. In that case you will get back your feedback in equally neat form.

Homework 2, due May 9

1. For a language $L \subseteq \mathbb{N}$, let

$$L^+ \stackrel{\text{def}}{=} \left\{ \underbrace{n\#n\#\ldots\#n}_{n \text{ times}} \,|\, n \in L \right\}$$

   (integers are represented in binary).

   Prove that if $L \in \mathsf{P}^{(L^+)}$ then $L \in \mathsf{P}$.

2. Prove that the following SUBSET SQUARE PROBLEM is NP-complete.

   INSTANCE: positive integers $\ell_1, \ldots, \ell_n$, written in binary.

   QUESTION: does there exist $S \subseteq [n]$ such that $\sum_{i \in S} \ell_i$ is a positive square number?

3. For a set of positive integers Λ, let $\Lambda\mathsf{P}$ be the class of languages L for which there exists a non-deterministic poly-time machine M such that

$$x \in L \equiv (\mathrm{acc}_M(x) \in \Lambda),$$

where $\mathrm{acc}_M(x)$ is the number of accepting paths when M is ran on x.

Describe those sets Λ for which there exists an oracle A such that $\Lambda\mathsf{P}^A \not\subseteq \mathsf{NP}^A$.

4. Prove that $co - \mathsf{NP} \subseteq \mathsf{PP}$.

5. Prove that testing if a given **undirected** graph is bi-partite is in NL.