Honors Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@math.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn16.html

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   Prove all of your answers.  If you work with others put their
names clearly at the top of the assignment.  Everyone must turn
in their own independently written solutions.  Homework is due at
the beginning of class unless submitted by e-mail as a PDF file
prepared from a TeX source.  Electronic submissions conforming to
these standards (no scans please) are accepted until Wednesday midnight
at lenacore@uchicago.edu.
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Homework 1, due October 12

1. Compute the sum

$$1 - 4 + 9 - 16 \ldots + (2n-1)^2 - (2n)^2$$

   as a closed form expression.

2. A function $f : \mathbb{N} \longrightarrow \mathbb{N}$ satisfies the following two properties:

$$f(n) = n - 5 \quad \text{if} \quad n > 25$$
$$f(n) = f(f(n+6)) \quad \text{if} \quad n \leq 25.$$

   What is $f(7)$?

3. Consider the following set of positive integers: $\{1, 2, 4, 8, \ldots, 1024, 3, 5, 9\}$ partially ordered by the divisibility relation $|$. How many different linear extensions does it have?

4. Simplify two ordinal expressions $(\omega + 2) \cdot \omega$ and $(2 + \omega) \cdot \omega$.

5. The *least common multiple* lcm(a, b) is defined as the smallest positive integer that is divisible by both a and b.

 Prove the identity

$$\gcd(\operatorname{lcm}(a, b), c) = \operatorname{lcm}(\gcd(a, c), \gcd(b, c)).$$