Honors Discrete Mathematics

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Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please) are accepted until Wednesday midnight at lenacore@uchicago.edu.

Homework 2, due October 19

1. Consider two recurrences $S(n)$, $T(n)$ such that $S(1) = T(1)$ and for $n > 1$,

$$\begin{aligned} S(n) &= \min_{1 \le m \le n-1} (S(m) + S(n-m)) - 1, \\ T(n) &= \max_{1 \le m \le n-1} (T(m) + T(n-m)) - 1. \end{aligned}$$

   Prove that $S(n) = T(n)$.

2. Show that the solution of $T(n) = T(n-1) + \frac{\lg n}{n}$ is $\Theta((\lg n)^2)$.

3. Find polynomials $x(n), y(n)$ such that

$$x(n)(n^2 + n - 1) + y(n)(n^2 + 1) = n.$$

4. A *monomial* (in two variables) is an expression of the form $m = X^d Y^e$ $(d, e \ge 0)$, and a monomial m *divides* another monomial m' if there exists a monomial m'' such that $m' = mm''$.

Prove that among any infinitely many monomials $\{m_1, m_2, \ldots, m_t, \ldots\}$ (in two variables) there always exist two monomials m_i and m_j $(i \neq j)$ such that m_i divides m_j.