Honors Discrete Mathematics

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Autumn Quarter, 2016

Prove all of your answers. In order to qualify for full credit, the answer in all numerical problems must be a closed-form expression in which only the factorials $n!$ with $n \geq 7$ are allowed. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please) are accepted until Wednesday midnight at lenacore@uchicago.edu.

Homework 5, due November 9

1. How many ways are there to choose five cards out of a standard deck of 52 cards in such a way that no suit is represented more than twice in the selection?

2. Prove the inequality

$$S(m, n-1) \leq \binom{n}{2} S(m, n) \ (2 \leq n \leq m).$$

3. How many ways of distributing 32 chess pieces into four distinguishable pouches are there? Pieces of the same kind and color (like two black rooks) are considered indistinguishable; the pouches are allowed to be empty.

4. Prove that $p_n(m)$ (the number of partitions of m using *at most* n numbers) is also equal to the number of partitions of $m + n$ using *exactly* n numbers.

5. Let $m = \frac{n^2+n+4}{2}$, and assume that $S_1, S_2, \ldots, S_m \subseteq [n]$ are pairwise distinct. Prove that there exist $1 \leq i < j \leq m$ such that $|S_i \cap S_j| \geq 2$.