Honors Discrete Mathematics

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn17.html

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Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.

Homework is due at the beginning of class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight by Chris at csj@uchicago.edu.

Homework 1, due October 11

1. Show how to compute $\gcd(1861, 2017)$ using the Eucledian algorithm.

2. Prove that
$$\gcd(ab, c) \leq \gcd(a, c)\gcd(b, c)$$
for all positive integers a, b, c.

3. (**challenge problem**) Find explicit constants $\epsilon, C > 0$ such that
$$\epsilon\sqrt{n} \leq \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n+1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)} \leq C\sqrt{n}$$
for all positive integers n.

   Note. You are not forbidden to use methods other than the induction, but in that case the absence of either explicit constants ϵ, C or a rigorous proof that they work for all values of n will be considered a very serious drawback.

4. Consider the set $\{2, 3, 30, 42, 2310, 2730\}$ partially ordered by divisibility: $m \preceq n$ if and only if $m|n$. How many different linear extensions does it have?

5. Let $(S, \leq)$ be a well-ordered set, X its infinite subset and $n \in \mathbb{N}$. Prove that X has the nth least element (i.e, $a \in X$ such that there are precisely $n-1$ elements $b \in X$ with $b < a$).