Honors Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.`

`Homework is due at the beginning of class` *`unless`* `submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight at the Canvas.`

Homework 6, due November 15

1. You throw a fair die n times. What is the probability that the *product* of the outcomes is divisible by:

   (a) 6?

   (b) 10?

   (c) 12?

   `Note. In order to qualify for full credit, the answer must be a close form expression.`

2. **(challenge problem)** Let $n \geq 1$ be an integer. Construct n events $E_1, \ldots, E_n$ in the same sample space such that $p(E_{i_1} \wedge \ldots \wedge E_{i_k}) = p(E_{i_1}) \cdot \ldots \cdot p(E_{i_k})$ for any $1 \leq i_1 < i_2 < \ldots < i_k \leq n$ with $k < n$ but $E_1, \ldots, E_n$ are not mutually independent.

3. Our class has 48 registered students, and the class next door has 40. Every student in every of the two classes attends with probability 80%, independently of all others.

   Looking for his dad, my son opens one of the two doors at random and sees 30 students in the room. What is the Bayesian probability that he opened the right door? Give both a close form exprassion (you may leave binomial coefficients unexpanded) and an estimate in the floating-point notation ($s \times 10^d$, $d \in \mathbb{Z}$, $1 \leq s < 10$) within the accuracy of two decimal places (for this part you may use calculators of course).

4. A fair balanced coin is tossed repeatedly until the first time we see the combination HEAD TAIL HEAD (in this order, but *not* necessarily consecutively). Calculate the distribution of the number of tosses, i.e. give a close form expression for the probability that their number in this experiment is equal to k for any integer $k \geq 3$.