Honors Discrete Mathematics

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn18.html

Autumn Quarter, 2018

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   Prove all of your answers with reasonable degree of mathematical
rigor.  If you work with others put their names clearly at the top
of the assignment.  Everyone must turn in their own independently
written solutions.  Shopping for solutions on the Internet is strongly
discouraged:  if you do it nonetheless, you *must* cite your source
and, as the very least, explain the solution in your own words.
   Homework is due at the beginning of Wednesday class unless submitted
by e-mail as a PDF file prepared from a TeX source.  Electronic
submissions conforming to these standards (no scans please!)  are
encouraged and accepted until Wednesday midnight by Leo at lenacore@uchicago.edu.
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Homework 5, due November 14

1. Prove the inequality
$$S(m, n-1) \leq \binom{n}{2} S(m, n) \ (2 \leq n \leq m).$$

2. How many ways are there to choose five cards out of a standard deck of 52 cards in such a way that all four suits are represented in the selection?

3. How many solutions are there to the inequality
$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 50,$$
where x_i are **odd** positive integers.

4. Let $q_k(m)$ be the number of ways to distribute m indistinguishable balls into indistinguishable boxes such that every non-empty box contains at least k balls. Prove the identity

$$q_k(m) = q_{k+1}(m) + q_k(m-k).$$

5. Let

$$m > \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{k},$$

and assume that $S_1, S_2, \ldots, S_m \subseteq [n]$ are pairwise distinct. Prove that there exist $1 \leq i < j \leq m$ such that $|S_i \cap S_j| \geq k$.