Honors Discrete Mathematics

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn18.html

Autumn Quarter, 2018

Prove all of your answers with reasonable degree of mathematical rigor. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.

Homework is due at the beginning of Wednesday class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight by Leo at lenacore@uchicago.edu.

Homework 6, due November 21

1. Six points are chosen inside the closed unit disk. Prove that there are two of them at the distance at most 1 from each other.

2. Let $\boldsymbol{A}, \boldsymbol{B} \subseteq [n]$ be two random subsets picked uniformly and independently of each other. Calculate $p(\boldsymbol{A} \cap \boldsymbol{B} = \emptyset)$ as a close form expression.

3. A fair die is rolled 2018 times. Calculate the probability that the total is divisible by 6 as a close form expression.

4. Give an example of four events A, B, C, D in the same sample space such that every three of them are mutually independent while $A \wedge B$ and $C \wedge D$ are not independent.

5. A fair balanced coin is tossed repeatedly until we see three heads (*not* necessarily consecutively). Calculate the distribution of the number of tosses, i.e. give a close form expression for the probability that their number in this experiment is equal to k for any integer $k \geq 3$.