Honors Discrete Mathematics

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn18.html

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Prove all of your answers with reasonable degree of mathematical rigor. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.

Homework is due at the beginning of Wednesday class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight by Leo at lenacore@uchicago.edu.

Homework 7, due November 28

1. Let X be picked uniformly at random from the set $[2018]$ and Y be picked uniformly at random from the set $\{2, 3, 6\}$, independently of X. Compute the expectation $E(X/Y)$ of their fraction.

2. The *mean deviation* $MD(X)$ of a random variable X is defined as $E(|X - c|)$, where $c = E(X)$ is the expectation of X (we briefly discussed this notion in the class, and denounced it).

   (a) Prove that for any two random variables X and Y on the same sample space, $MD(X + Y) \leq MD(X) + MD(Y)$.

   (b) Prove that if X and Y are additionally known to be independent, then this inequality is *always* strict, unless one of the variables X, Y is trivial (that is, takes one fixed value with probability 1).