Honors Discrete Mathematics

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Prove all of your answers with reasonable degree of mathematical
rigor.  If you work with others put their names clearly at the top
of the assignment.  Everyone must turn in their own independently
written solutions.  Shopping for solutions on the Internet is strongly
discouraged:  if you do it nonetheless, you *must* cite your source
and, as the very least, explain the solution in your own words.
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Homework 6, due November 18

1. We simultaneously roll a standard (cubical) die, an icosahedral one (with 20 facets numbered from 1 to 20) and a dodecahedral one (numbered 1-12). Prove that the probability to get a total ≤ 15 is equal to the probability to get a total of ≥ 26.

2. Prove or disprove the following.

   Let E, F, G be three events in the same sample space such that $0 < p(F), p(G) < 1$ and, moreover, $p(E|F) \geq p(E)$ and $p(F|G) \geq p(F)$. Then $p(E|G) \geq p(E)$.

3. Alice plays a slot machine until her first win, then keeps playing until the first loss and then goes home. Compute the expectation of the number of plays as a closed form expression if the probability of winning in every round is p, where $0 < p < 1$.

4. Let X, Y be two independent random variables with values in $\mathbb{Z}_n$ and, moreover, assume that X has uniform distribution. What can we say about the distribution of $X + Y$?

5. Let X have the binomial distribution $B_{100,1/2}$ and Y be uniformly distributed in $\{2, 3, 6\}$. Assuming that X and Y are independent, compute $E\left(\frac{X}{Y}\right)$.

`Note.` To qualify for full credit, the answer must be a plain rational number.