Honors Discrete Mathematics

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   Prove all of your answers with reasonable degree of mathematical
rigor (feel free to ask us when in doubt).  If you work with others
put their names clearly at the top of the assignment, everyone must
turn in their own independently written solutions.  Shopping for
solutions on the Internet is strongly discouraged.  But if you accidentally
stumble across it, then it will be accepted as long as you cite
the source and explain the solution in your own words.
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Homework 2, due October 19

1. Consider the standard ordering $(\mathbb{Q}_{>0}, \leq)$ of positive rational numbers and its induced sub-ordering $(\mathbb{Q}_{(0,1)}, \leq)$ on the set of all rational numbers strictly between 0 and 1. A general theorem by Cantor implies that they are isomorphic. Provide an **explicit** isomorphism between them.

2. Prove that $\omega^2 \neq \omega \cdot 2$.

3. Prove that if $2^n - 1$ is a prime then n is a prime, too.

4. Let $x_1, \ldots, x_n$ be positive integers. Prove that

$$\gcd(x_1, x_2)\gcd(x_2, x_3) \cdots \gcd(x_{n-1}, x_n)\gcd(x_n, x_1) \leq x_1 x_2 \cdots x_n.$$

5. Assume that $a, b \in \mathbb{N}$ are different positive integers. Prove that the set

$$\left\{ \gcd(n^2 + an + 1, n^2 + bn + 1) \mid n \in \mathbb{N} \right\}$$

is finite.